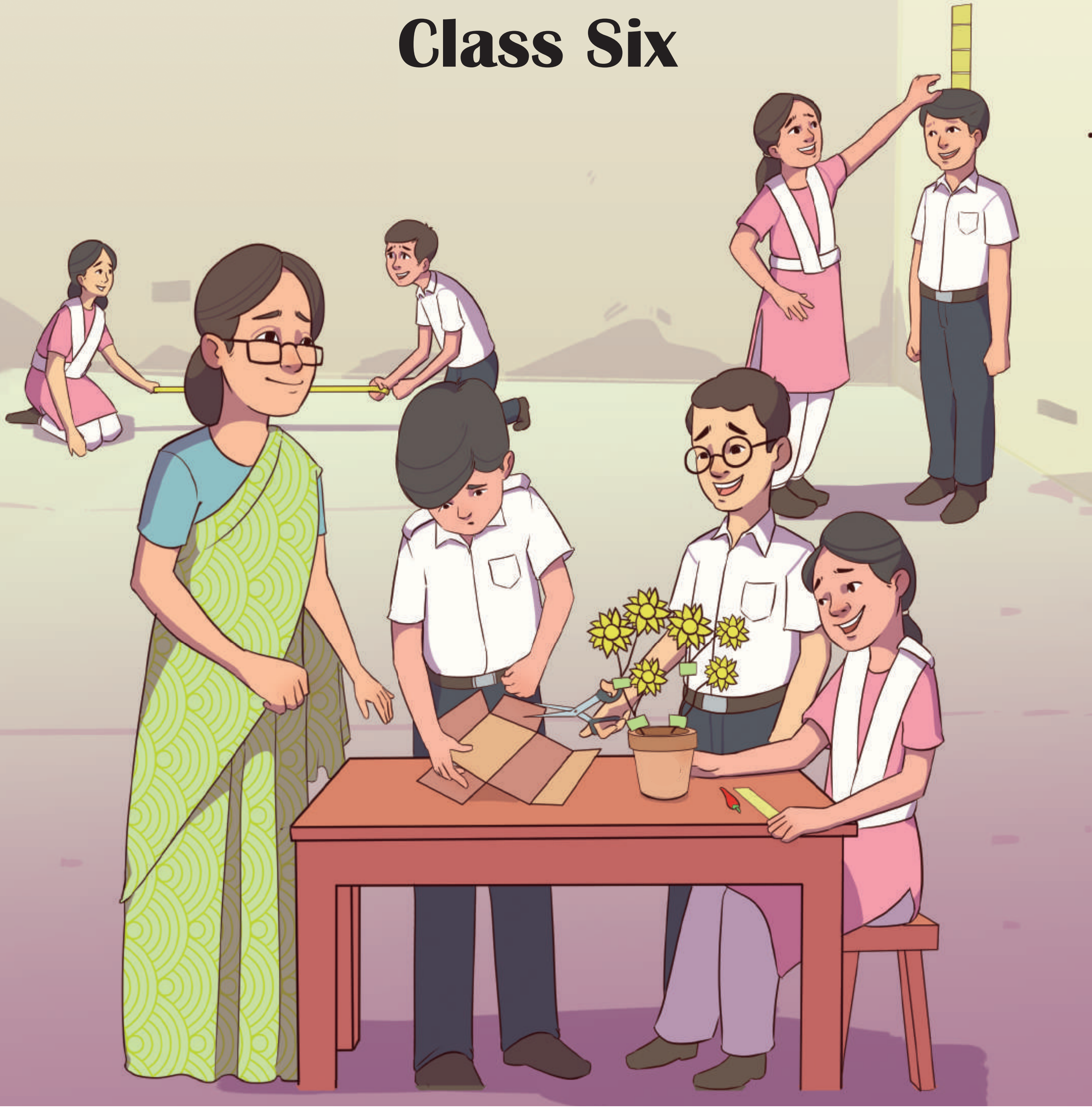
Mathematics

Class Six



National Curriculum and Textbook Board

১৯৭৪ সালে জাতিসংঘের অধিবেশন আলোকিত করে বাংলায় প্রথম বক্তব্য রাখেন 'সর্বকালের সর্বশ্রেষ্ঠ বাঙালি বঙ্গবন্ধু শেখ মুজিবুর রহমান'

জাতির পিতা বঙ্গবন্ধু শেখ মুজিবুর রহমান এর সুযোগ্য কন্যা বাংলাদেশের বর্তমান মাননীয় প্রধানমন্ত্রী শেখ হাসিনা জাতিসংঘে বাংলায় ভাষণ প্রদান করেন

১৯৭৪ সালের ২৫শে সেপ্টেম্বর জাতিসংঘের সাধারণ পরিষদের অধিবেশন আলোকিত করে বাংলায় প্রথম ভাষণে সর্বকালের সর্বশ্রেষ্ঠ বাঙালি বঙ্গবন্ধু শেখ মুজিবুর রহমান বলেন -'বাংলাদেশের মতো যেই সব দেশ দীর্ঘ সংগ্রাম ও আত্মদানের মাধ্যমে নিজেদের প্রতিষ্ঠিত করিয়াছে, কেবল তাহাদেরই এই দৃঢ়তা ও মনোবল রহিয়াছে, মনে রাখিবেন সভাপতি, আমার বাঙালি জাতি চরম দুঃখ ভোগ করিতে পারে, কিন্তু মরিবে না, টিকিয়া থাকিবার চ্যালেঞ্জ মোকাবেলায় আমার জনগণের দৃঢ়তাই আমাদের প্রধান শক্তি ।'

Developed by the National Curriculum and Textbook Board as a textbook according to the National Curriculum 2022 for Class Six from the academic year 2023

Mathematics

Class VI

(Experimental Version)

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Preface

In this ever-changing world, the concept of livelihood is altering every moment. The advancement of technology, in accordance with knowledge and skill, has accelerated the pace of change. There is no alternative to adapting to this fast changing world. The reason is, the development of technology is at its zenith compared to any time in the human history. In the fourth industrial revolution era, the advancement of artificial intelligence has brought a drastic change in our employment and lifestyles and this will make the relationship among people more and more intimate. Varied employment opportunities will be created in near future which we cannot even predict at this moment. We need to take preparation right now so that we can adapt ourselves to that upcoming future.

Although a huge economic development has taken place throughout the world, the problems of climate change, air pollution, migrations and ethnic violence have become much more intense than before. The epidemics like COVID 19 has appeared and obstructed the normal lifestyle and economic growth of the world. Different challenges and opportunities have been added to our daily life.

Standing on the verge of these challenges and possibilities, implementation of sustainable and effective solutions is required for the transformation of our large population into a resource. It entails global citizens with knowledge, skill, values, vision, positive attitude, sensitivity, capability to adapt, humanity and patriotism. Amidst all these, Bangladesh has graduated into a developing nation from the underdeveloped periphery and is continuously trying to achieve the desired goals in order to become a developed country by 2041. Education is one of the pivotal instruments to attain the goals and there is no alternative to the modernization of our education system. Developing an effective and updated curriculum has become crucial for this modernization.

Developing and revising the curriculum is a regular and vital activity of National Curriculum and Textbook Board. The last revision of the curriculum was done in 2012. Since then, a lot of time has passed. The necessity of curriculum revision and development has emerged. For this purpose, various research and technical exercises were conducted under the supervision of NCTB during the year 2017 to 2019 to analyze the prevalent situation of education and assess the learning needs. Based on the researches and technical exercises, a competency-based incessant curriculum from K-12 has been developed to create a competent generation to survive in the new world situation.

In the light of the competency based curriculum, the textbooks have been prepared for all streams (General, Madrasah and Vocational) of learners for grade VI. The authentic experience driven contents of this textbook were developed in such a way that teaching learning becomes comprehensible and full of merriment. This will connect textbooks with various life related phenomenon and events that are constantly taking place around us. We hope that learning will be profound and life-long now.

Issues like gender, ethnicity, religion, caste, the disadvantaged and students with special needs have been taken into special consideration while developing the textbook. I would like to thank all who have put their best efforts in writing, editing, illustrating and publishing the textbook.

If any one finds any errors or inconsistencies in this experimental version and has any suggestions for improving its quality, we kindly ask them to let us know.

Professor Md. Farhadul Islam
Chairman
National Curriculum & Textbook Board, Bangladesh

Dear Students,

Heartiest welcome to the joyful learning experience of mathematics. The National Curriculum and Textbook Board (NCTB), Bangladesh has taken initiative to introduce new textbooks for all the students of grade six. The mathematics textbook is one of those. You will observe that two main aspects were taken into consideration while designing this book for all of you and those are to solve mathematical problems by observing and analyzing the surroundings and to create opportunity for applying mathematical calculation and estimation skills in problem solving in the daily life.

This mathematics textbook is consisted of twelve learning experiences. In each of those learning experiences, the topics are introduced in a way that it will ensure your active participation as well as joyful learning. You will participate in different group work/pair work to complete the activities. Your learning will end when you will be able to apply those learning in your daily life. In this learning process the textbook will be considered as an enabling material.

The teachers will facilitate and guide your learning thoroughly in this process. We hope that this journey will empower you to achieve mathematical skills successfully and will guide your journey in the language of mathematics. We are also hoping that this book will help you to become more inquisitive to explore the world mathematics.

All the Best.

The Writers Panel

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The Story of Numbers

In our everyday life, we come across different kinds of numbers since waking up in the morning and till we go to sleep at night. Let us see the following pictures-

How could human beings know the different kinds of numbers that you are seeing now? Just think. How did they count and write numbers years ago?

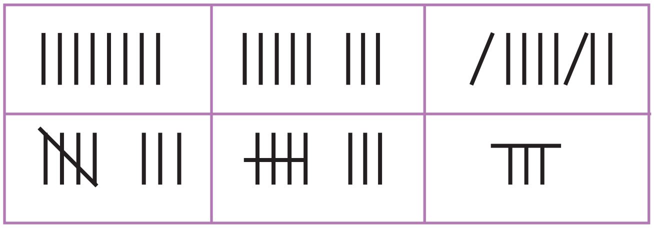
The answer to this question will be found in "The Story of Numbers." Let us now learn the funny story of how the numbers came. Let us go back to a few thousand years, when humans depended solely on hunting or forest fruits for food.

At that time they used to wake up in the morning hearing the chirping of the birds. Afterwards, may be they washed their faces in river water and would start looking for foods.

Is there any difference of counting and using numbers between our daily life and the daily life of humans who lived many years ago?

Let us now see a few examples of counting numbers of ancient people-how they counted numbers by tally marks, rope knots or by using stones.

Counting by Tally Marks

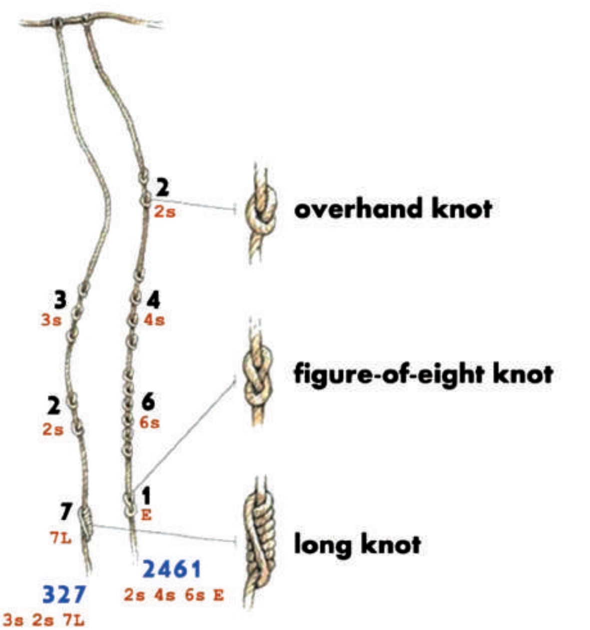


Human beings of different decades expressed number 8 in different kinds of tally marks as shown above. Can you mention any other way like this to express number 8?

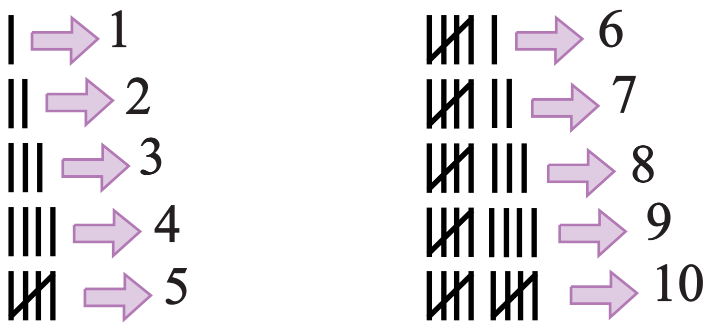
Counting with Rope Knots

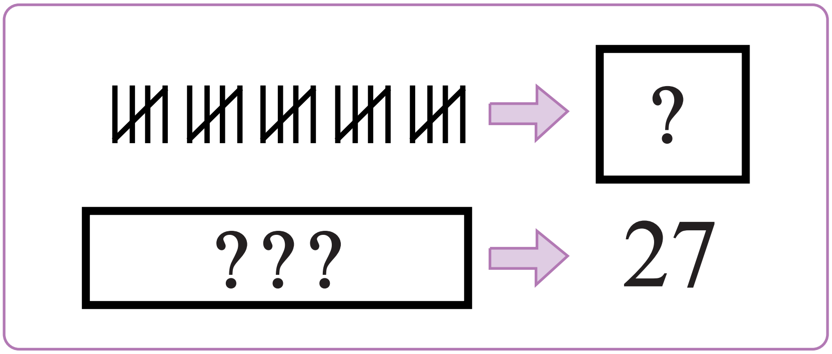
Do you know that humans expressed numbers with rope knots?

Do you understand anything from the picture below?



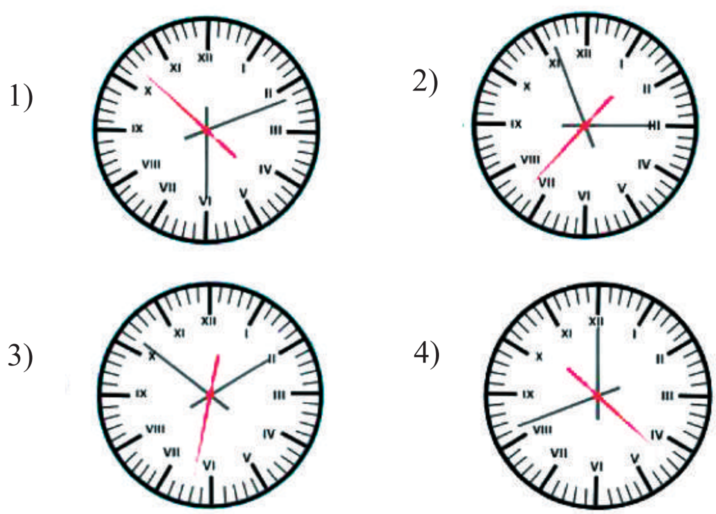
Counting with Tally Marks





Understanding Time on the Clock

Which watch shows what time?



Fill up the following table

Number	How is it written on the clock	Number	How is it written on the clock
1		7	
2		8	
3		9	
4		10	
5		11	
6		12	

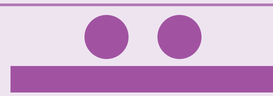
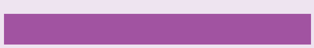
Exercise

Now try writing the numbers 13, 20 and 67 in the same method of numbers shown on the clock?

Puzzle

Do you know how the Mayans used to express numbers?

Can you fill up the following table?

Our Known Numbers	Mayan expression	Our Known Numbers	Mayan expression
0		6	
1		7	
2		8	?
3	?	10	
4		14	?
5		19	?

The Story of Decimal Number System

Mathematician Aryabhata (pronounced as Arjovotto) of Indian subcontinent introduced a method of creating numbers by using 0,1,2,3,4,5,6,7,8,9 these ten signs. This method is called Decimal Number System.

Let's learn how Aryabhata thought and introduced this system.

Aryabhata thought, 'If I want to express numbers then I'll express as shown below:'

0	1	2	3	4	5	6	7	8	9

Then he thought, 'All the signs I have are already used once. We see that each of the numbers is increasing by 1 like the Roman System. That means if 1 is added with 1 then we will get 2, again if 1 is added with 2 then we will get 3. Now, if I continue writing numbers then what will I write after 9?'

1st Writing Completed:

0	1	2	3	4	5	6	7	8	9
0 ??									

'But I've used all the signs once and that has to be written in numbers. Where will I be writing that?'

0	1	2	3	4	5	6	7	8	9
10									

'I've written zero to show that all the numbers are written once and written 1 at the left of each number to express that those numbers are already written once.' He further said, 'Only 1 and the 1 used before 0 in 10, doesn't mean the same. That means, their value is not equal. Because, the 1 before 0 in 10 tells that we've written all the numbers once. Now what will happen if we continue writing numbers, after writing all the numbers once, as shown below?

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29

Here the values of 2 in 20 and 2in 12 or only 2 are not same. We write XX in Roman Numbers to mean 10 added with 10 resulting in 20 but, in our system if we write 10 we don't mean 1 added with 0 resulting 1. Aryabhatta didn't name the numbers so far. He just wanted to express how many times he had been sign number. How long would he be able to write like this?

So, if we use the entire 9 signs twice in different manner then we can write up to 99. At this point, again Aryabhatta got stuck on what to do next. He thought of writing the numbers again following the same system. That means we have to start from 0 and that has to be explained.

99	99	99
----	----	----
0	00	100

Now if we focus on the left sided numbers written above, we will see that we've already written each of the numbers 9 times. Therefore, I've to put a zero.

Now, if we write 1 at the left side then,

The 1 at the left of 100 means how many times the numbers appearing at the left of the double digit numbers are written. But then, what do we call the numbers appearing at the right of double digit numbers? At this stage, Aryabhatta gave them names. He named the left sided number appearing in double digit numbers as Tens (Doshok) and the leftmost number appearing in three digit numbers as Hundreds (Shotok). That means, 10 is the ten times multiple of 1 and 100 is the ten times multiple of 10.

From here he noticed a wonderful thing that, 'If I keep writing the numbers side by side and move from one digit place to the other, the number increases ten times. Now, we learnt that with three numbers we can write up to 999 and if we want to write more after that we just have to increase one more number to the left. Similarly, every time the digit place changes, the number increases 10 times. The decimal system is thus introduced from the thought of calculation. Now, if we see that a number increases 10 times, every time the digit place is changed then that is called number system. Have to add something

So, there are 10 signs or symbols in our number system and we call them Onko in Bengali and Digit in English.

We're seeing numbers from 1-9 and they express something which means they got value. But 0 only has no value therefore 0 has to be with other numbers. This is why 0 is called auxiliary number and the numbers from 1-9 are called significant numbers. From our discussion of Roman Numbers we already know that writing numbers side by side like XX or XC is called Notation. If we want to write a number then we have to use the signs from 0-9 in a systematic way. According to the system, every time a sign appears in the left, the formed number will be increased 10 times.

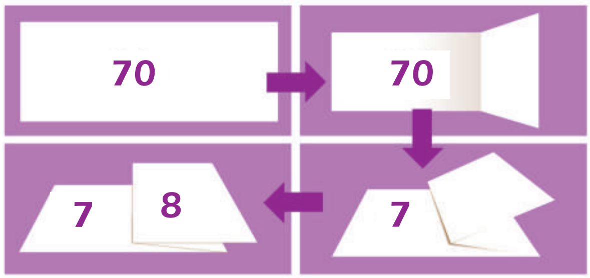
Now, let's see,
123 is a number
Here, there are three digits. From right to left Aryabhatta named them ones, tens and hundreds.

1	2	3
Hundreds	Tens	Ones

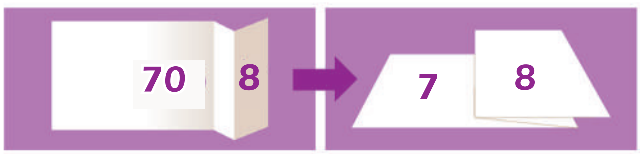
We have to read it like: 1 Hunderds, 2 Tens 3 Ones
The real value of the number will be:
One Hundreds (100) + Two Tens (20) + Three Ones (3) = One hundred and twenty three (123).
Just like this we've started writing numbers and as a result we got Decimal Number System.

Place Values hidden in Paper Folds

Method of Folding

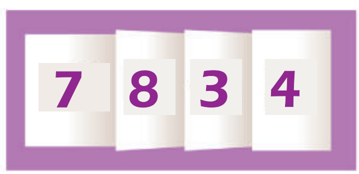


How to look at Place Values?



How not to look at Place Values?

Let's create larger numbers



Local System

	Lacs		Thousands		Hundreds	Tens	Ones
Crore	Millions (Nijut)	Lac	Tens of Thousands (Ajut)	Thousand			
Eighth	Seventh	Sixth	Fifth	Fourth	Third	Second	First
1	3	0	8	2	5	2	4

One crore Thirty lacs Eighty Two thousand Five hundred Twenty Four

International System

Billions			Millions			Thousands			Hundreds	Tens	Ones
Twelfth	Eleventh	Tenth	Ninth	Eighth	Seventh	Sixth	Fifth	Fourth	Third	Second	First
2	4	4	2	1	3	0	8	2	5	2	4

Two hundred and forty four billion two hundred and thirteen million eighty thousand five hundred and twenty four

Comparison between Local and International Systems

					Lacs		Thousands		Hundreds	Tens	Ones
?	?	?	?	Crore	Millions (Nijut)	Lac	Tens of Thousands (Ajut)	Thousand			
Twelfth	Eleventh	Tenth	Ninth	Eighth	Seventh	Sixth	Fifth	Fourth	Third	Second	First
3	4	7	8	1	9	9	3	5	6	1	8
Billions			Millions			Thousands			Hundreds	Tens	Ones

Express the above number in both Local and International System.

Is there anything above Crore in Local system?

You have to find it out. You can discuss it with all - teachers, guardians, relatives and friends.

Pair Work

- In each pair, make a total of 16 pieces of paper by writing 0, 1, 2,……9 digits with repetition. A sample is given below:

0	1	1	2
3	5	6	7
4	8	8	9
2	6	0	8

- Now, each of the members of the pair has to take 8 pieces of paper through lottery from the 16 pieces of paper made earlier.
- Then, each student of the pair will arrange his 8 pieces of paper from the lottery, forming highest and lowest possible numbers. They will write them down in their exercise book.
- Now, between the highest numbers written by the members of the pair, the student whose highest number is greater will get 1 point. And the other one will get 0 point.

- Now, between the smallest numbers written by the members of the pair, the student whose lowest number is smaller will get 1 point. And the other one will get 0 point.
- The student having more points will be the winner; if the points are equal then the game will be declared as a draw.

Exercise

1) Without any repetition, form the highest possible number and smallest possible number of four digits using the digits given below.

   a) 2, 8, 7, 4 b) 9, 7, 4, 1 c) 4, 7, 5, 0 d) 1, 7, 6, 2 e) 5, 4, 0, 2

   (Hint: 0754 is a three digit number)

2) Form the highest and smallest number using any of the digits twice.

   a) 3, 8, 7 b) 9, 0, 5 c) 0, 4, 9 d) 8, 5, 1

   (Hint: Think about all the terms about using a digit twice)

3) Form the highest and smallest possible numbers by using four different digits given below and by fulfilling all the terms stated below. (The first one is solved for you)

a) Digit 7 has to be in the place of Ones.

Largest	9	8	6	7
Smallest	1	0	2	7

(The number must not begin with 0. Why?)

b) Digit 4 has to be in place of Tens.

Largest			4	
Smallest			4	

c) Digit 9 has to be in place of Hundreds.

Largest		9		
Smallest		9		

d) Digit 1 has to be in place of Thousands.

Largest	1			
Smallest	1			

Puzzle

There is a birthday gift for you in the box below. But the problem is the box is locked. Just below the lock the digits from 0 to 9 are written. For opening the lock you need a secret three digit number. Different features of that secret number are mentioned in the paper below.

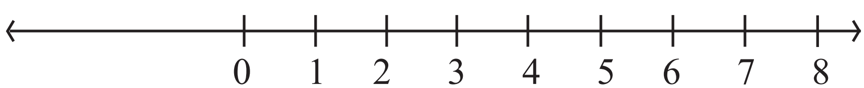
Now, find out the secret number and win gifts.

6	8	2	One digit is correct and the digit is in right place.
6	1	4	One digit is correct but the digit is in wrong place.
2	0	6	Two digits are correct but the digits are in wrong place.
7	3	8	None of the digits are correct.
7	8	0	One digit is correct but the digit is in wrong place.

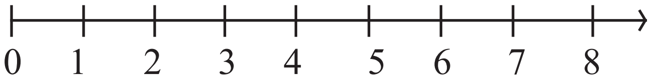
Concepts of the four processes of Integers through Number Lines

Number Lines

- Draw a straight line and mark any point on the line with 0.
- Mark the second point at the right side of 0 with 1.
- The distance between the points marked with 0 and 1 is called Unit Distance.
- Now, mark a point with 2 on this straight line at the right side of 1 and at a unit distance.
- In the same way, mark 3, 4, 5 …. on the straight line maintaining unit distance.
- Through this system you will be able to mark 0 at the right side, and all the Integers greater than 0.



Here, we will discuss about the 0 and its right side part on the above number



What is the distance between 2 & 4 here? Definitely, it's 2 units. Will you be able to tell the distance between 2 & 6, 2 & 7 respectively?

In the number line you will see that number 7 comes at the right side of number 4. This 7 is greater than 4 that means $7 > 4$. Also, number 8 is at the right side of number 6 therefore $8 > 6$.

These observations help us saying that, between two Integers, the number appearing at the right side is the greater number. We can also say that, the left sided Integer is the smaller number.

Example: $4 < 9$; 4 is at the left of 9. Similarly, $12 > 5$; 12 is at the right of 5.

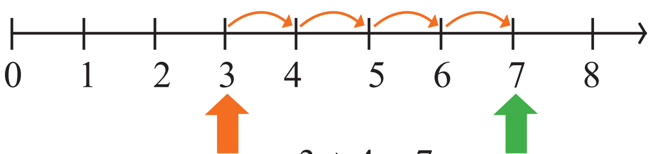
Now you give your opinion about 10 & 20.

Mark the integers 30, 12, 18 on a Number Line. Which number comes at the farthest left side? Can you tell which number between 1005 & 9756 will be on the right side of the other?

On a Number Line, mark the Integers appearing after 12 and before 7.

Addition on the Number Line

The addition of Integers could be shown on Number line. Let's see the addition of 3 & 4.



$3 + 4 = 7$

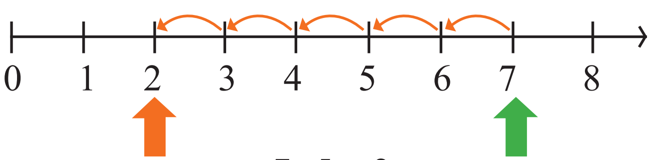
Let's start from 3. As we're to add 4 with it, give 4 jumps on the right; 3 to 4, 4 to 5, 5 to 6 and 6 to 7 (As shown in the above picture). The last position of 4 jumps has to be on 7.

So, the addition of 3 & 4 will be 7. That means, $3+4=7$

Mark the sums of 4+5, 2+6, 3+5 and 1+6 using the Number Line.

Subtraction on the Number Line

The subtraction of two Integers can also be shown on Number Line. Let's find out 7-5.



$7 – 5 = 2$

Let's start from 7. As we're going to subtract 5 from the number, therefore it will go to its left side, 1 unit in 1 jump. Following this system it would reach point 2 in 5 jumps.

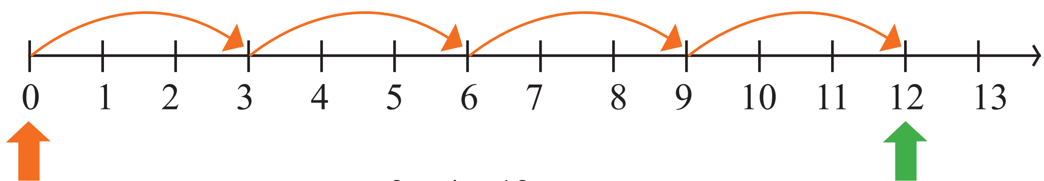
So, the subtraction of 7 & 5 will be 2. That means, 7-5=2

Mark the subtractions of 8-3, 6-2 and 9-6 using a Number Line.

Multiplication by Number Line

The multiplication of Integers can be seen on the Number Line.

Let's find out 4×3 using Number Line.



$3 \times 4 = 12$

Start from 0 Go 3 jumps in the unit at your right. You need to give 4 jumps like this.

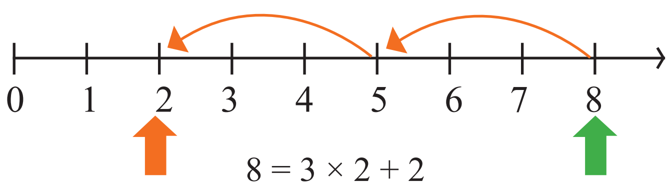
Where will you reach? At 12.

So, we say that, 3×4=12.

Determine the multiplication of 6×2, 6×7 and 5×3 using the Number Line.

The Concept of Division by Number Line

We have seen addition, subtraction and multiplication on Number Line. Now, we will see the concept of division. Division means subtracting divisor from dividend again and again. And finally, when we reach a smaller number than divisor, we call it the remainder.



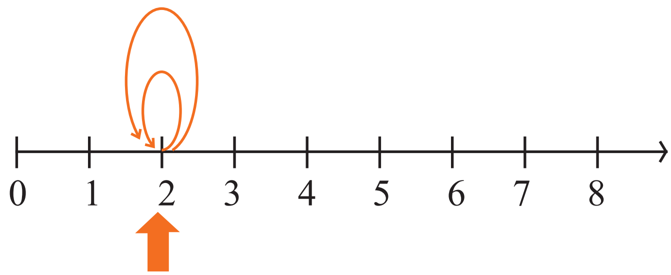
$8 = 3 \times 2 + 2$

Dividend= 8, Divisor = 3,

Quotient = 2, Remainder = 2

Now, you divide 13 by 4 using Number Line and find out the Quotient and Remainder.

Let's divide 2 by 0 using Number Line.



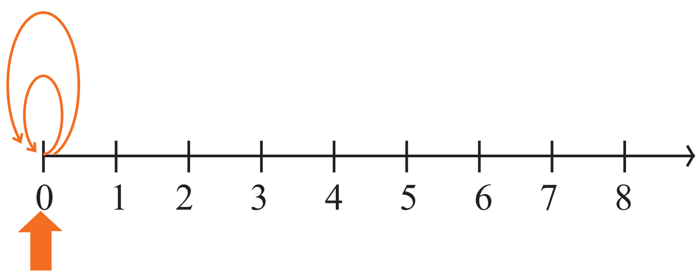
Here, dividend = 2 and divisor = 0. As a result, a jump of 0 length from 2 if attempted a number of times, that means every time 0 is subtracted, the position will be 2. So this subtraction will never end. As a result the dividing process will keep continuing. No quotient will be found. That means, no quotient is found as per the definition of division process.

For this-

We call the process of dividing 2 with 0 as Undefined.

Similarly, we will call the process of dividing 1, 3, 4, 5, 6, 12 and similar numbers with 0 as undefined.

But, what will be the result if 0 is divided by 0



This time something different has happened.

Here, dividend = 0 and divisor = 0. Therefore, a jump of 0 length from 0 if attempted a number of times, that means every time 0 is subtracted, the position will be 0. Even if no jump is attempted, that means if 0 is not subtracted even for once, the same thing will happen. Therefore, the quotient can be anything like 0, 1, 2, 3, 8, 15, 16. In this case, determining one definite quotient is not possible.

This is why- we call the process of dividing 0 with 0 as Indeterminate.

Divisibility

Concept of Divisibility

If we get 0 as the remainder while dividing one Integer by another Integer then it is said that the first Integer (Dividend) is divisible without remainder by the second Integer (Divisor).

Find out whether 12 is divisible without remainder by 1, 2,3,4,5,6 & 7 in conventional method or with the help of Number Line.

The rule of divisibility by 2 & 4 and explaining the reasons with place values Divisible by 2

If we write a few multipliers of 2 then we get,

$2\times0 = 0$, $2\times1 = 2$, $2\times2 = 4$, $2\times3 = 6$, $2\times4 = 8$

$2\times5 = 10$, $2\times6 = 12$, $2\times7 = 14$, $2\times8 = 16$, $2\times9 = 18$ etc.

Let us see the process of getting the product. If any number is multiplied by 2 then 0, 2, 4, 6 or 8 will be in the place of Ones in the product. So, it is seen that, if any number has 0, 2, 4, 6 or 8 in place of Ones then the number will be divisible by 2. Now, let's check whether our observation is true or not.

When 3516 is written as per place values:

3 5 1 6

The place value of 6 =6

The place value of 1= $1\times10=1\times2\times5$

The place value of 5= $5\times100=5\times2\times50$

The place value of 3= $3\times1000=3\times2\times500$

$3516 = 3000 + 500 + 10 + 6$

Here, digit in Ones is 6, that is divisible by 2. Moreover, all the place values of all the digits coming at the left side of Ones are divisible by 2.

That means if the digit in the place of Ones is divisible by 2 then the number will also be divisible by 2.

We call such numbers as Even Numbers.

When 3517 is written as per place values:

3 5 1 7

The place value of 7 = 7

The place value of 1=1×10=1×2×5

The place value of 5=5×100=5×2×50

The place value of 3=3 ×1000=3×2×500

3517 = 3000 + 500 + 10 + 7

Here, digit in Ones is 7 which is not divisible by 2. But, all the place values of all the digits coming at the left side of Ones are divisible by 2.

That means if the digit in the place of Ones is not divisible by 2 then the number will also not be divisible by 2.

We know such numbers as Odd Numbers.

If a number has zero or even number in place of Ones then that number will be divisible by 2.

Divisible by 4

When 3512 is written as per place values:

3 5 1 2

The place value of 2 = 2

The place value of 1 = 1 × 10

The place value of 5 = 5×100 = 5×4×25

The place value of 3 = 3 ×1000 = 3×4×250

3512 = 3000+500+10+2

Here, 10 is not divisible by 4. But, all the place values of all the digits coming at the left side of Tens are divisible by 4.

Again, 3512=3000+500+12

Here, 12 is divisible by 4. So, the number 3512 is divisible by 4. Since the number formed with the digits of Ones and Tens is divisible by 4, the whole number is also divisible by 4.

If a number is formed with the digits of Ones and Tens of a given number and if the formed number is divisible by 4 then the given one will also be divisible by 4.

Or, if the digits at Ones & Tens are zero, the number will be divisible by 4.

Group Work: Using the place values, present and explain the reasons of the rule of divisibility by 8.

Divisible by 5

Let's write a few multipliers of 5.

$5\times 0 = 0$, $5\times 1 = 5$, $5\times 2=10$, $5\times 3 = 15$, $5\times 4 = 20$, $5\times 5 = 25$, $5\times 6 = 30$,

$5\times 7 = 35$, $5\times 8 = 40$, $5\times 9 = 45$ etc.

Let us see the process of getting the product. If any number is multiplied by 5 then the digit in the place of Ones will be 0 or 5. So, it is seen that, if there is 0 or 5 in place of Ones then the number will be divisible by 5.

Individual Work: Using the place values, present and explain the reasons the rule of divisibility by 5.

The rule of divisibility by 3, 6, 9 using place values with presentation and explanation of the reasons: Divisible by 3

1 4 7

The place value of $7 = 7$

The place value of $4 = 4\times 10 = 4 \times (9+1)$

$= 4\times 9+4\times 1 = 4\times 3\times 3+4$

The place value of $1= 1\times 100=1\times (99+1)$

$= 1\times 99+1\times 1=1\times 3\times 33+1$

Here, $4\times 3\times 3$ and $1\times 3\times 33$ numbers are divisible by 3 and the sum of the digits at Ones, Tens & Hundreds $= 1+4+7=12$; which is divisible by 3.

So, the number 147 is divisible by 3.

Again, let's consider the number 148.

1 4 8

The place value of 8= 8

The place value of 4= $4\times10=4\times(9+1)$

$= 4\times9+4\times1=4\times3\times3+4$

The place value of 1= $1\times100=1\times(99+1)$

$=1\times99+1\times1=1\times3\times33+1$

Here, $4\times3\times3$ and $1\times3\times33$ numbers are divisible by 3. But, the sum of digits in place of Ones, Tens & Hundreds= $1+4+8=13$; is not divisible by 3.

So, the number 148 is not divisible by 3.

If the sum of all digits of a given number is divisible by 3, then the given number is also divisible by 3.

Divisible by 6

If a number is divisible by 2 and 3 then the number will be divisible by 6.

Divisible by 9

Let's consider the number 378.

3 7 8

The place value of 8=8

The place value of 7 = $7\times10 = 7\times(9+1)$

$= 7\times9+7\times1 = 7\times9+7$

The place value of 3=$3\times100 = 3\times(99+1)$

$= 3\times99+3\times1 = 3\times9\times11+3$

Here, 7×9 and $3\times9\times11$ each is divisible by 9 and the sum of the digits at the place of Ones, Tens and Hundreds= $3+7+8=18$, which is divisible by 9. So, the number 378 is divisible by 9.

If the sum of all the digits of a given number is divisible by 9 then the given number will be divisible by 9.

Group Work: Finding out easy rules of divisibility by 11

Divisibility by 11

308, 1331 and 61809 - all these numbers are divisible by 11.

Let us see whether we can find any easy pattern of divisibility by 11 using the following table.

Numbers	Sum of the digits from right side (Digits positioned in Odd serials)	Sum of the digits from right side (Digits positioned in Even serials)	Difference
308	8+3=11	0	11-0=11
1331	1+3=4	3+1=4	4-4=0
61809	9+8+6=23	0+1=1	23-1=22

Magic of Three Cards

- Take a paper and tear it into 8 pieces. After that, write the numbers from1 to 8 on those papers.

1	2	3	4
5	6	7	8

- Select any three pieces of paper from the eight pieces.

(Example)

Selected Number Cards

2	6	3

- Take the three numbers from the selected three pieces of paper and form the highest and smallest possible numbers. Now, subtract the smallest number from the highest number.

(Example)

$$
\begin{array}{rccc l}
 & \boxed{6} & \boxed{3} & \boxed{2} & \text{Highest Number of three digits} \\
- & \boxed{2} & \boxed{3} & \boxed{6} & \text{Smallest Number of three digits} \\
\hline
 & 3 & 9 & 6 &
\end{array}
$$

- Now, it's time for magic.
- Each of you will take turns to tell the teacher about the digit you have in the place of Ones of your subtraction result.

  (In the above shown example, the digit in the place of Ones will be = 6)
- Tell the other two digits situated in the places of Tens and Hundreds to your teacher.
- Will you be able to show such magic as your teacher? Try on your own to find the trick behind this magic.

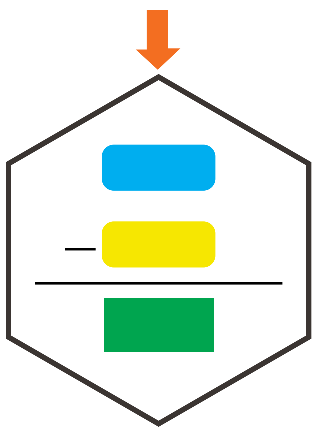
Show this magic to your friends.

Show it to your family members, relatives and neighbours too.

Find out the age through favourite name

$10 \times$ Your Age =

$9 \times$ Total letter numbers in your favourite person's name =



Tell the number you got in green portion to your teacher. Then the teacher will be telling you, your age.

Show this magic to your friends.

Show it to your family members, relatives and neighbours too.

The Story of Two Dimensional Objects

Greek Scholar Euclid

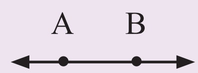
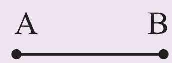
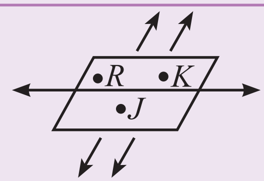
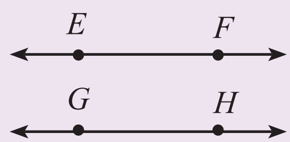
Geometry is an ancient but interesting branch of mathematics. We could measure our playground, garden, house, lands just because we know geometry. You must be curious to know what geometry means? It is known that in Greece, people meant lands by 'Geo' and measurement by ' Metron'. These 'Geo' and 'Metron' together formed Geometry, which we say as Jamiti (Geometry) in Bengali. Now you might ask, why was this geometry even necessary?

Many years ago, civilizations were built depending on agriculture. For agriculture, lands were necessary. And for measuring the lands, geometry was necessary. But nowadays, geometry is used not only to measure the lands but also to solve many critical problems of mathematics. The usage of geometry is also found in ancient Egyptian, Babylonian, Indian, Chinese and South American Inca civilizations.

But the perfect and organized form of geometry is clearly seen only in Greek Civilization. Greek scholar Euclid wrote his famous book "Elements" organizing the formulas of geometry. Moreover, Thales, Pythagoras, Plato, Ptolemy, Archimedes and many more mathematicians contributed in developing geometry.

Basic Concepts of Geometry

Let's look at the following table and fill in the blank portions.

Geometric Name	Description	Figure	How to read
Point	Point has no length, height and width.	• A	Point
Line	Line has no definite length.		Line
Segment	Segment has definite length.		Segment
Ray	Ray has a starting point but has no definite length.		Ray
Face	Face has only length and width. Face is two-dimensional.		Face RJK
Parallel lines	Two parallel lines on the same face never intersect.		EF & GH lines are parallel
Angle			
Adjacent Angle			
Right Angle			
↓			
↓			

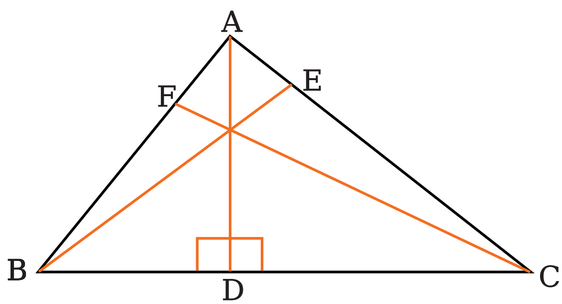
Triangle of Paper

Make a few triangles of paper. Now, attach or draw the triangles on your exercise book. Prepare a table as given below and fill it up.

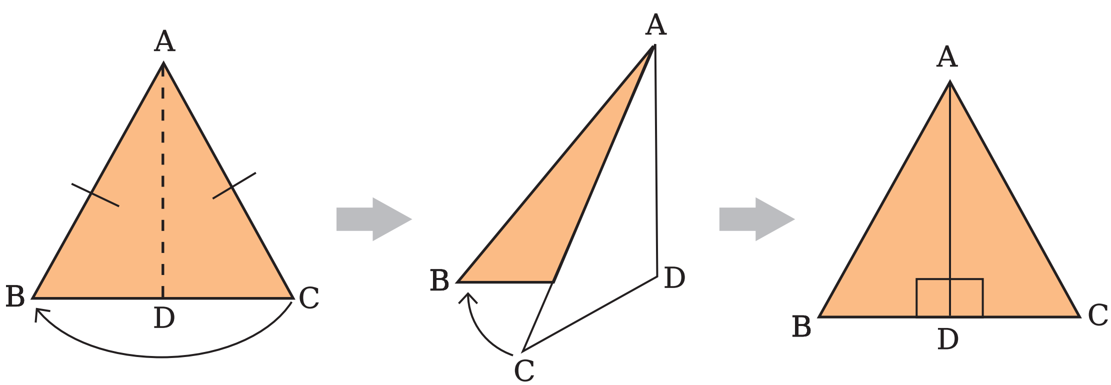
Figure	1st Angle	2nd Angle	3rd Angle	Sum of the three Angles	Length of the 1st Side	Length of the 2ndSide	Length of the 3rdSide	Type of Triangle

All the sides of a triangle can be the face and as per that there can be three heights as well

Acute Angle has three heights (Have some Correcions)

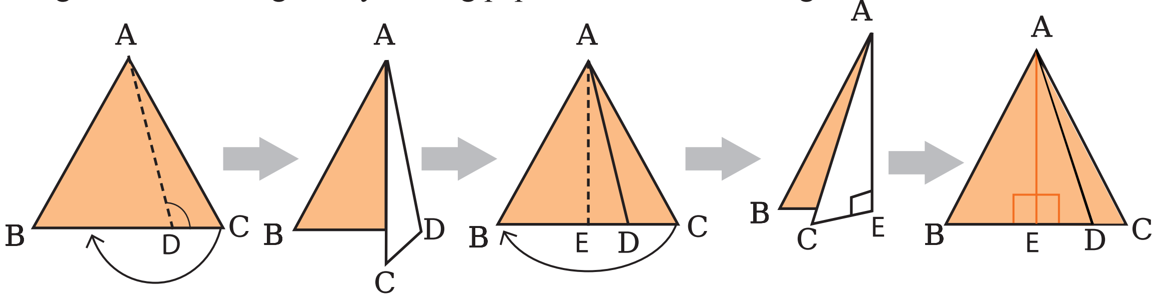


Height of Right Angle (Have some Correcions)

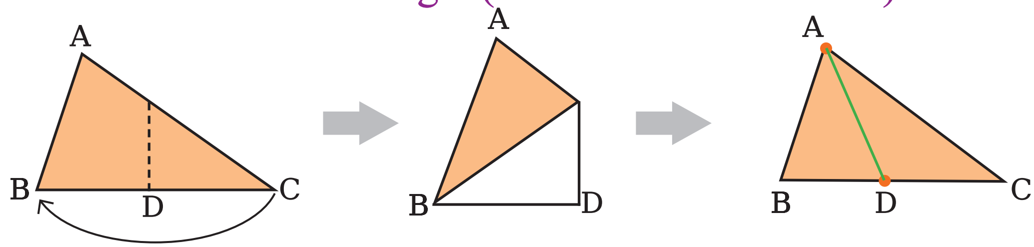


Height of an obtuse angled triangle (Have some Correcions)

In this figure, △ ABD is Acute Triangle and △ ACD is Obtuse Triangle. AE is the height of both Triangles. By folding paper show the other heights.



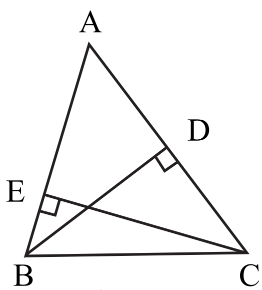
Find the median of a triangle (Have some Correcions)



The green straight line connects the vertex of a triangle with the midpoint of the opposite side. Hence we call it 'the Median of the triangle'.

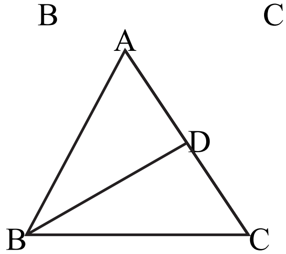
Exercise

1)



In the figure if AB = 100 cm, AC = 120 cm and BD = 80 cm, then CE =?

2)

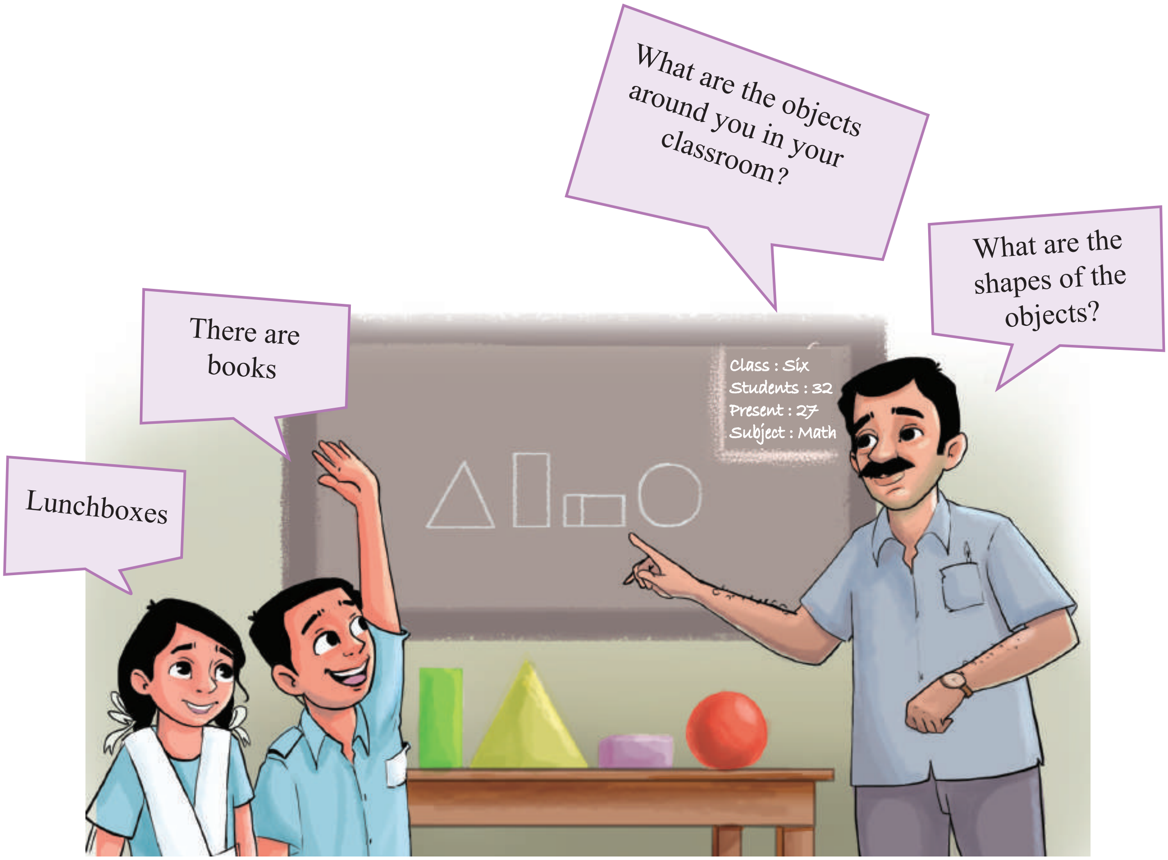


In the figure, BD is the Median of the triangle ABC and the length of the side BC is twice the length of AD.
What is the type of the triangle? Show reasons for your answer.

3) The lengths of the three sides of a right-angled triangle are 5 cm, 12 cm and 13 cm.
   a) Draw a proportionate diagram.
   b) Find the length of the perpendicular drawn from the right-angled vertex to the opposite side.

Let's find objects of different shapes

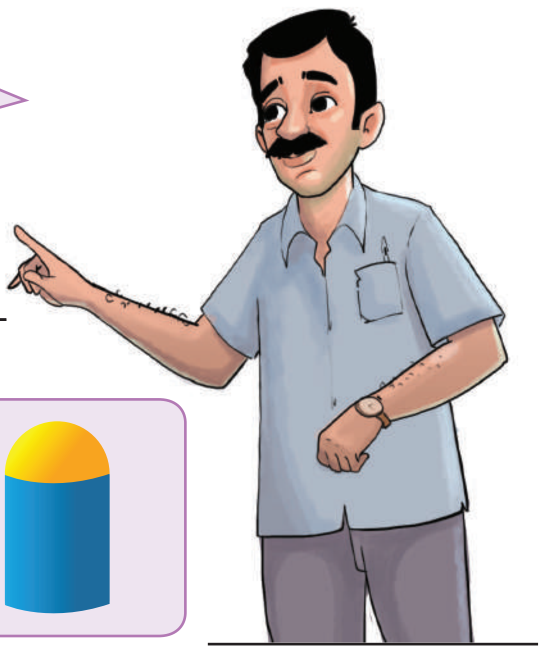
There are objects of different shapes around us. All the objects do not appear same, their characteristics are also different. Today we shall learn about the shapes and characteristics of different objects and we shall look for the similarities/differences among them.



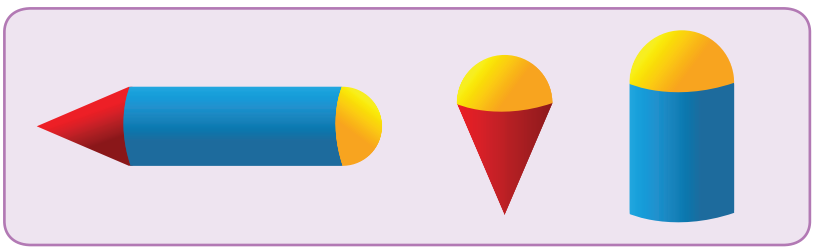
The objects of different shapes in your classroom. Think about the reasons for the differences of the shapes of the objects. Are there any differences among the planes of these objects? Are the numbers of sides of these objects around us different? Identify the angles, sides and planes of these by intensive observations.

Picture	Name	Sides	Angles	Planes	Name of the geometrical shape	Two Dimensional/Three Dimensional
	Book	12	24	6	Rectangular solid	three dimensional
						
						
						
						
						
						

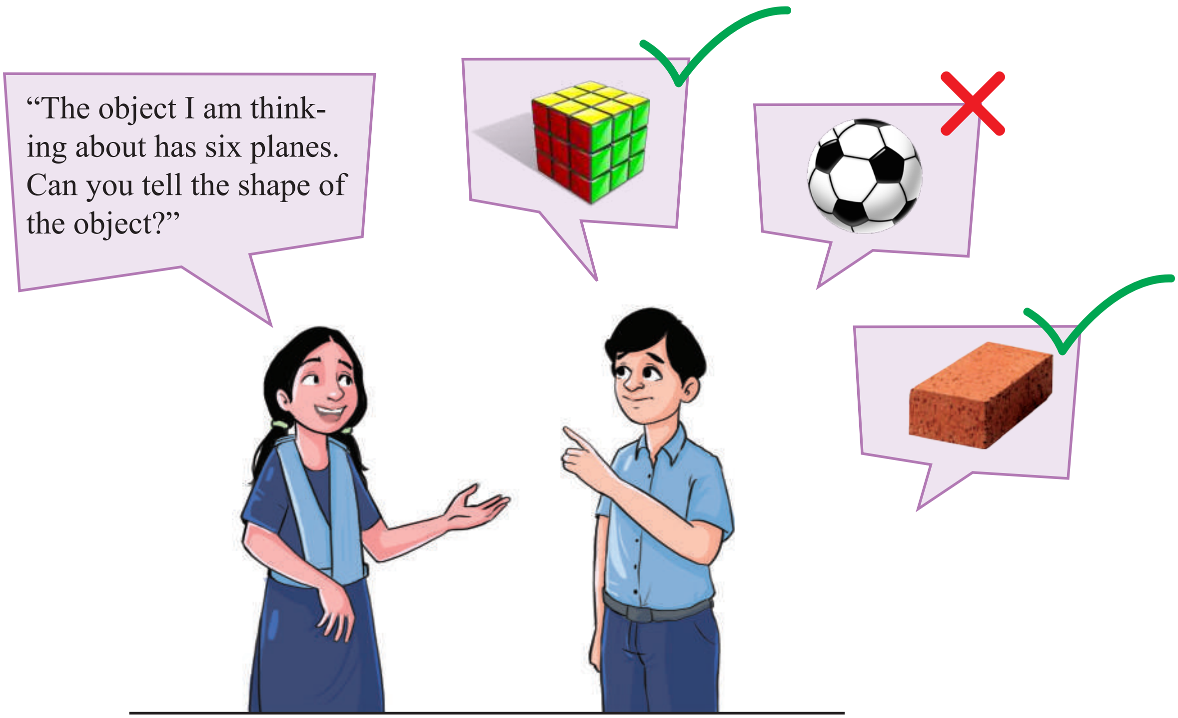
These objects are compound shaped. Have you ever seen such objects? Think and answer.



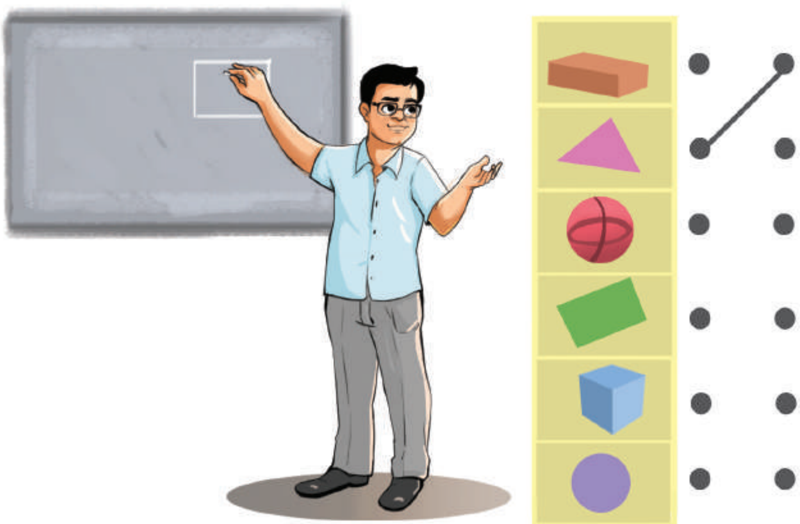
Now can you say which shape with which combines to make the following objects?



In the picture two students are discussing about Geometric shapes



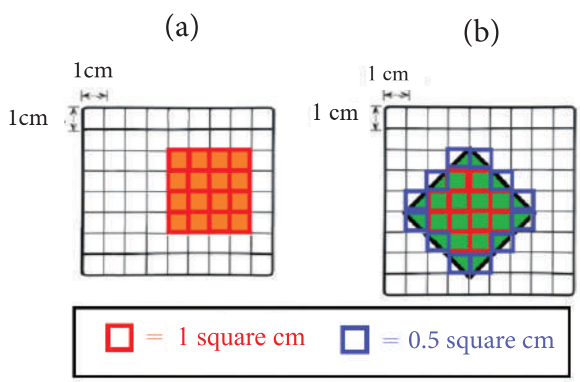
Connect the pictures on the left with the conditions on the right



- 3 sides and 1 plane
- 4 sides and 1 plane
- 6 planes and all planes equal
- 1 closed curve and 1 plane
- 6 planes but all are not equal
- 1 curved surface

There are different objects around us. We can measure them using different methods. Sometimes we use a ruler. Again, sometimes we use grid/graph to measure objects of different shapes.

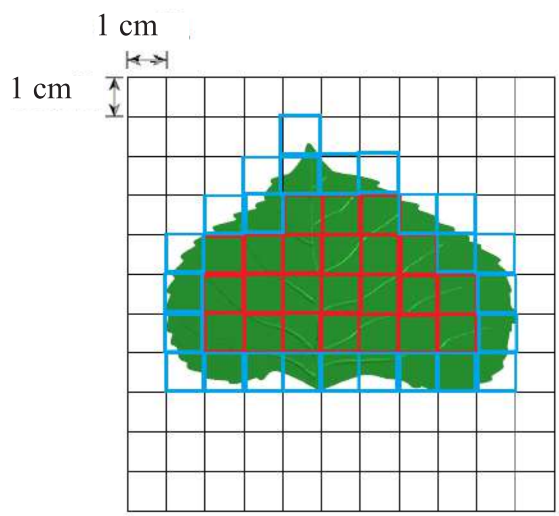
Technique to measure a Quadrilateral using grid

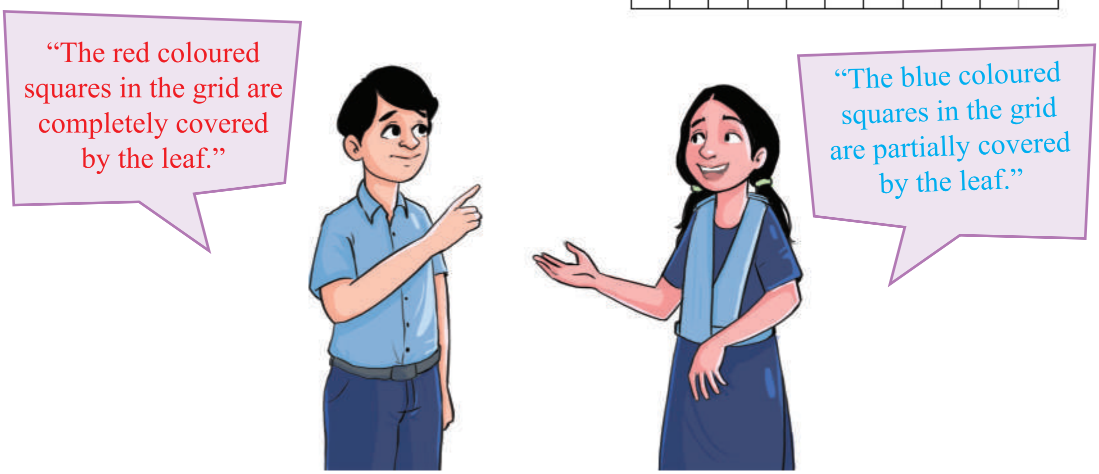


- (A) In the first figure (A), area of 16 red marked squares = 16 x 1 cm2 = 16 cm2. There is no square marked blue in it. So, there is no chance that the measurement will be more or less.
- (B) In the second figure (B), area of 12 red marked squares = 12 x 1 cm2 = 12 cm2. In figure (B) the area of 12 squares marked blue = 12 x 0.5 cm2 = 6 cm2
- → Area obtained from the grid = 12 cm2 + 6 cm2 = 18 cm2
- Here, each of the blue coloured squares has exact area 0.5 cm2
- Can you find the areas in figures (A), (B) using any other Geometric methods?
- So, are the areas obtained using the grids in figures (A) and (B) correct or exact, or near or approximate area?

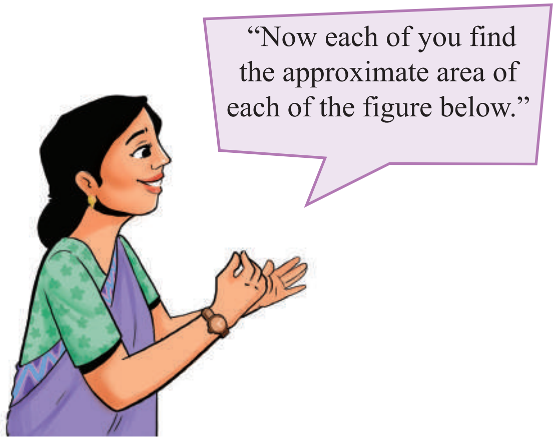
The teacher is exhibiting a leaf in the classroom. Observe it carefully. Think and determine a plan how you can measure the leaves.

The method to measure leaves in grid

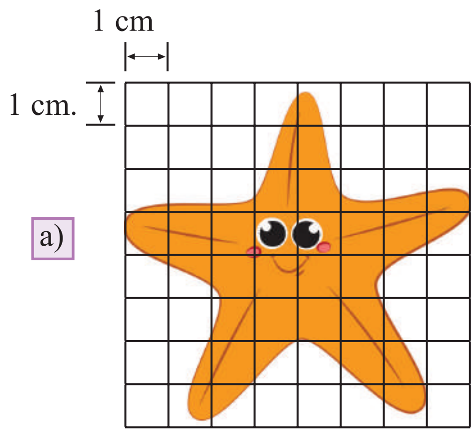




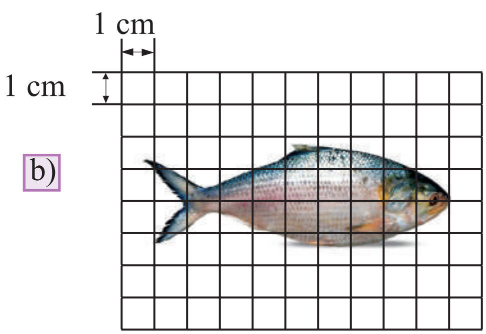
- → Area of the 23 red coloured squares = 23 x 1 cm2
  = 23 cm2
- → Area of the 25 blue coloured squares = 25 x 0.5 cm2
  = 12.5 cm2
- → Approximate area of the leaf = 23 cm2 + 12.5 cm2
  = 35.5 cm2
- But the areas of all the blue coloured squares are not exactly 0.5 cm2. So, is the area of the leaf obtained by the method of grids, exact or approximate?
- Now find the approximate area of the same leaf in the picture by taking length of each square of the grids as 2 cm and 0.5 cm respectively.
- According to you, which area of the leaf obtained will be nearer to the actual area of the leaf? Justify your answer.



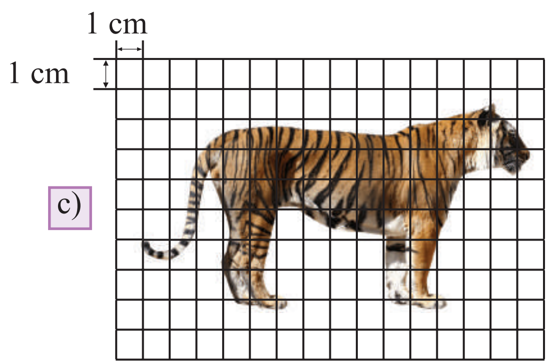
a)



b)



c)



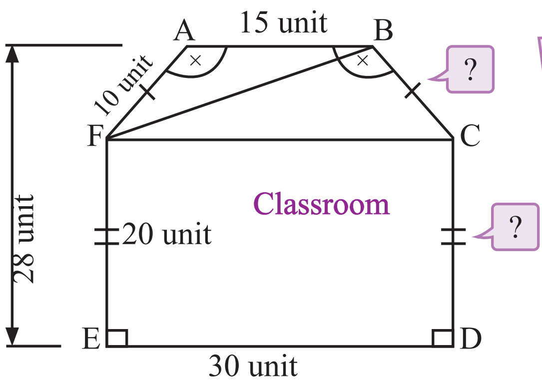
Group activities/task/work: How big is our Classroom?

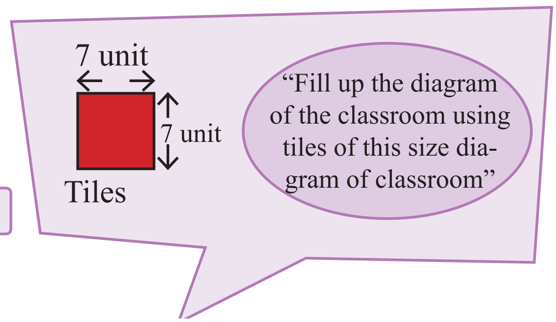
You will measure the wall and the floor of your classroom through this task. All of you in the group will perform the task by planning. Here you will complete the process of peer evaluations according to the instructions given by the teacher. Help your friends with special need (vision impaired or physically challenged) to take part in the group task.

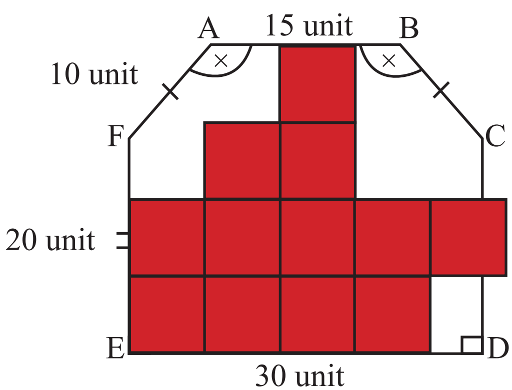
Worksheet: Let us measure the study room

What is the area of the floor of your study room?	
How many tiles may you possibly need for that floor? (including some extras) (choose your own size and design of the tiles)	
What is the area of the space inside the classroom including the ceiling that needs to be painted? (If necessary, get help to find the measurement and complete the task).	

Puzzle







a) How many tiles were needed to fill up diagram of the classroom?

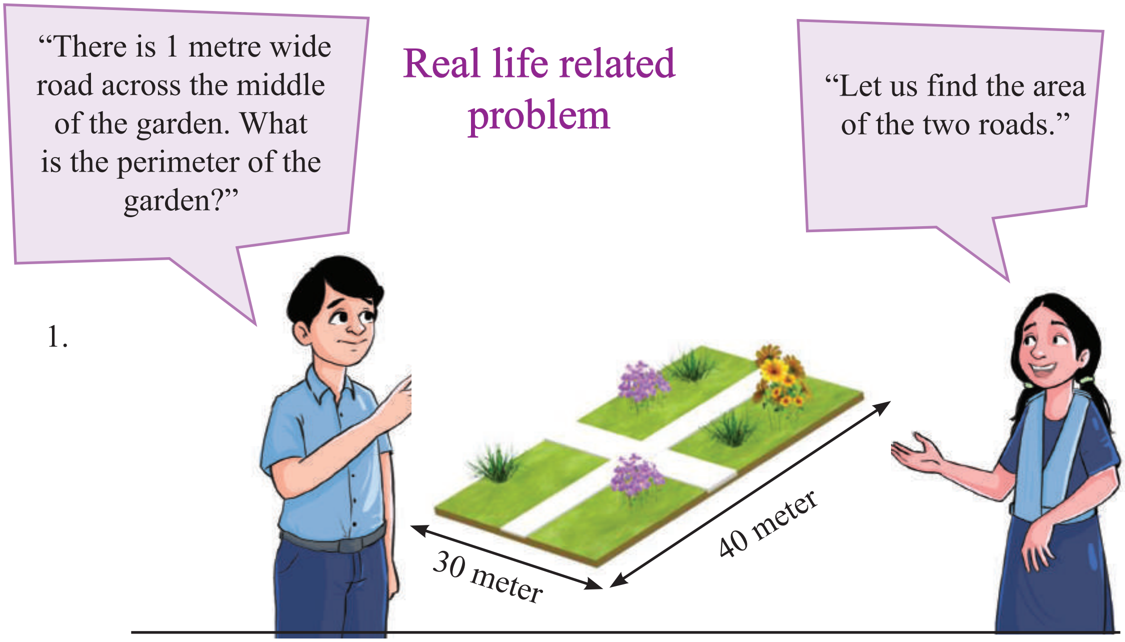
b) Calculate the number of tiles needed by finding the area of the classroom in the diagram and the area of a tile.

(Clue : AB and ED are parallal lines. You can draw the heights of $\triangle$ ABF and $\triangle$ BCF.)

c) Give a logical explanation if there is any difference between the results obtained in parts (a) and (b)

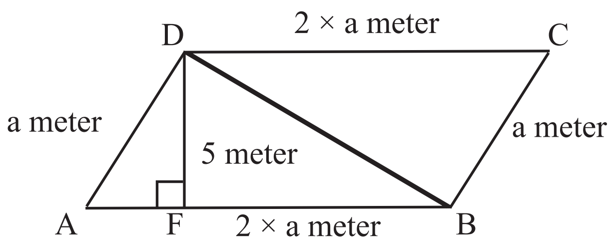
Real life related problem

1.



2. The area of a rectangular field is equal to the area of a square. The length of the rectangular field is 4 times the width. The cost of rope per metre is Tk 7. The cost of the rope to make fence twice around the field is Tk 5600.
   a) What will be the perimeter of the rectangular field?
   b) If you sow a papaya plant in every 4 m2 area how many papaya plants will be needed?

3.



In the diagram, the perimeter of the parallelogram field is 180 metre. The area of the parallelogram can be obtained in more than one way.

   a) Find the area of the parallelogram in more than one way with logical explanations.
   b) Show that the area of the parallelogram field = twice the area of the triangle ABD.

4. The length of the floor of a room is 26 metre and the width is 20 metre. How many mats of length 4 metre and width 2.5 meter will cover the whole floor? What will be the total cost if each mat costs Tk 45?

Sample rubrics for peer evaluation in Group task on measuring two dimensional objects

Each student will use this Rubric for peer evaluation for the members of his/her Group. Teachers will instruct the students how to conduct this evaluation process.

During group activity, observe the activity of members of your group and conduct the process of peer evaluation. If your classmate completes the whole work, then award three stars, if done partial work, then award two stars and if completed the measurement, but results are incorrect, then award star one star. Here if needed, you can take help from your teacher.

Completed fully	Done partially	Measured, but incorrect result	Did not participate in task
★ ★ ★	★ ★	★	⊘

Name of evaluating student :	Name of other members in group					
Field of evaluation	A	B	C	D	E	F
Successful in finding the area of the walls in the classroom						
Successful in finding the part of the wall to be painted						
Successfully found the area of the floor of the classroom						
Determined the number of tiles needed for the floor						
During group activity/task , discussed with the other members of group.						
During group activity helped everyone						
To determine the correct result, took measurement two/three times						

Comments:

Information Investigation and Analysis

We use diverse types of information in our daily life. The present era is known as the era of Information Technology. Living in this era of Information Technology it is our absolute necessity to know about information, information investigation and analysis and gathering the skill to apply them. It is particularly important to reach to a logical decision from several results obtained from the information analysis.

Information and Data

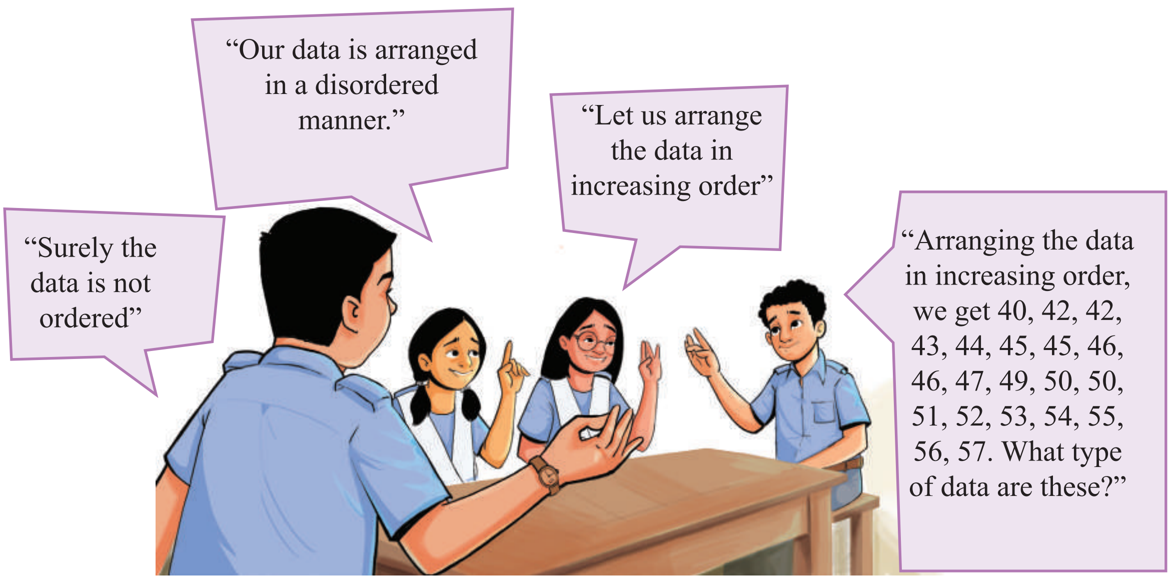
You must have noticed that your teacher takes your attendance in the classroom every day and keeps a record of your presence/absence. At the end of each test/examination in different subjects, they keep records of your marks obtained and they identify your weaknesses basing on these records. They also take appropriate steps to remedy them. At many a time, we go to the market and can directly collect the market prices of different commodities. Many of you have visited stadiums watch live football or cricket games. Many of you have gone to the zoo and learnt many things about different animals and birds. We can also find out different information from daily newspapers, radio, television etc about weather, games, commodity prices, health issues etc.

Data: You know that, in examination, marks are awarded in numerical values. When information is expressed and presented in numerical values, we get the data. For example, Ahona is 11 years old – this is information but the number 11 is a data.

Arranged and non-arranged data

There are 40 students in your class. Divide yourselves in two groups A and B. Weigh yourselves and write down your weights (in Kg) in your exercise books. Suppose the weights (in Kg) of the members in group A are as follows:

Group A: 45, 50, 42, 43, 56, 40, 46, 51, 55, 57, 44, 45, 50, 54, 53, 42, 46, 47, 52, 49

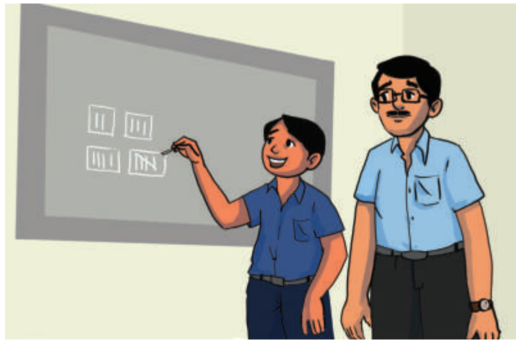


Collection, arrangement and presentation of data using a bar diagram are shown through a practical work:

Tally for Birth Month

Let’s fill up the following table for finding our birth month:

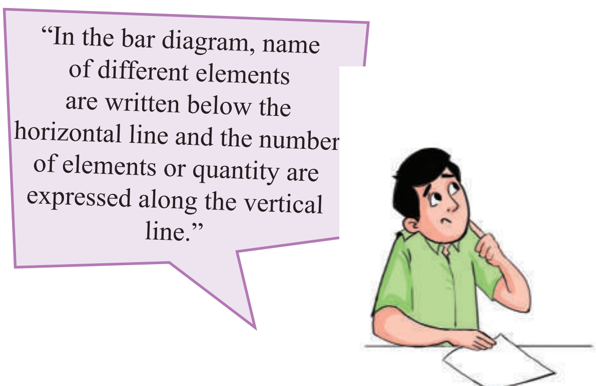
Month	Tally Marks	Total number of the Tally
January		
February		
March		
April		
May		
June		
July		
August		
September		
October		
November		
December		

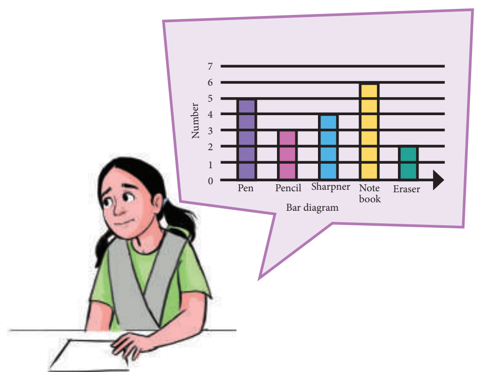


a) What does each Tally indicate?
b) In which month most of the students was born?
c) In which month least number of students was born?
d) Is there any relationship between the total number of Tally and the total number of students?

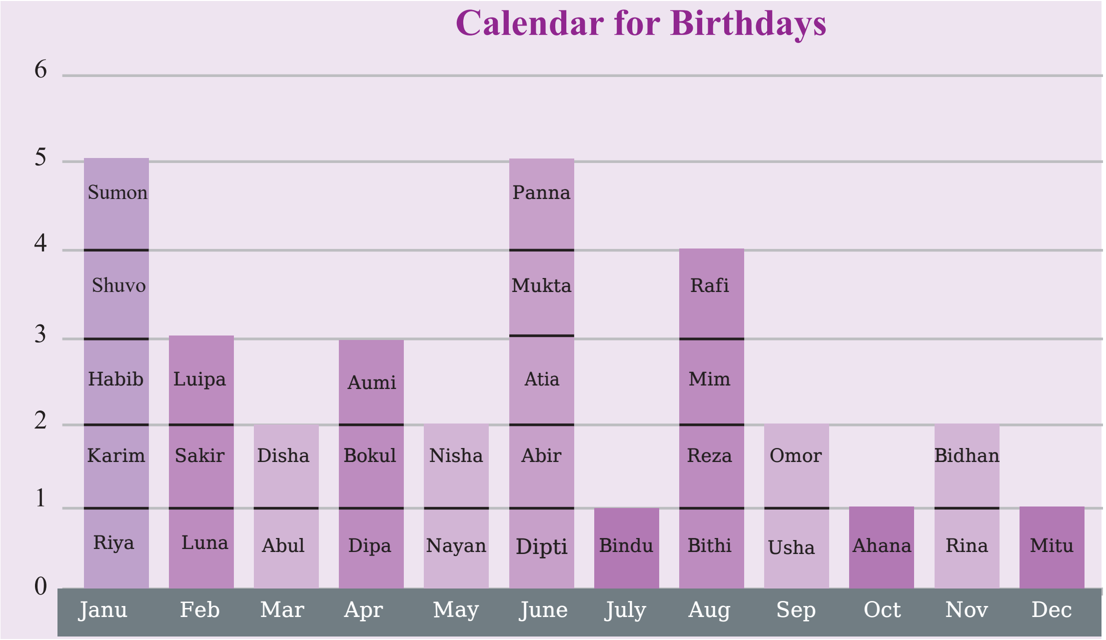
Total number in the Tally mark may be called the frequency

Now each of us shall use the table of birth month on the board to draw a bar diagram. You know about bar diagram from your previous classes. Using these diagrams, we can easily compare the data of different elements





Prepare a bar diagram like the sample below using same size small papers with your names written on them or using your (stamp size) pictures or using chart paper or back pages of old calendars. Arrange it according to your birth month.

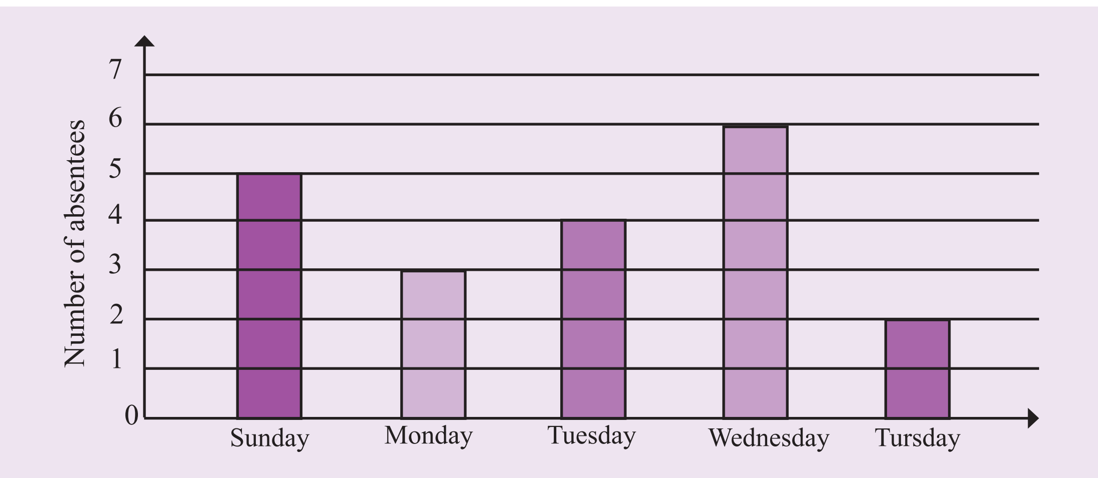


Presentation of Information using Bar Diagram

There are 40 students in your class. The number of students absent in the last week in your class is given in the following table:

Days	Sunday	Monday	Tuesday	Wednesday	Thursday
Number of absentees	5	3	4	6	2

Let us present the numbers of absentee students using a bar diagram.



Bar Diagram

In the above bar diagram 5 days of the week are shown along the horizontal line and the number of students absent on each of those days is shown along the vertical line.

Report of Individual work

Prepare a report by collecting 5/6 bar diagrams of the same kind from different sources (Daily Newspapers, magazines, Internet, Yearly reports of different Organizations, ….)

Table representing the Individual Work

Pictures of Bar	Sources of Pictures	Time	Brief Description	Comments

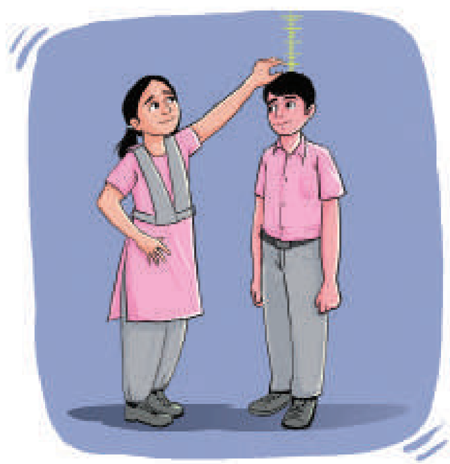
Mean

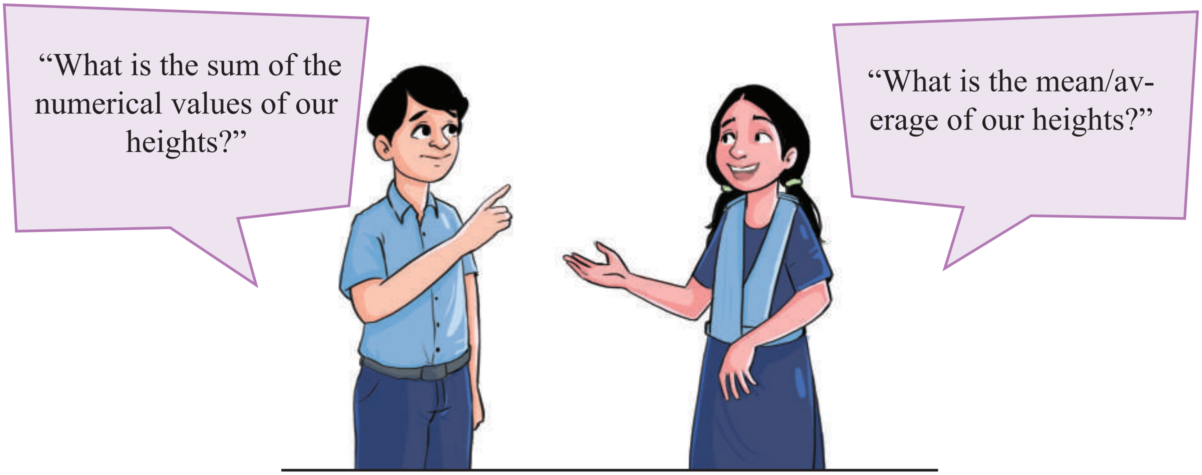
In Mathematics, Mean is a number which represents in general a collection of numbers or a data set. Some samples are collected and their total is divided by their number. That is, divide the sum of all the numerical values of the numbers by the total number of quantities in the Data. We see or hear a lot about the Mean in our daily lives. For example: our income per head, yearly average production of Hilsha, average number of wickets taken by a bowler per over in cricket games, mean number of students present in class, etc.

Measure the height (in cm) of each individual and find the mean/average height

Table: Complete the following list by measuring individual heights (in cm) and find the mean height:

Serial number	Height (cm)	Serial number	Height(cm)
1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	





a. Sum of the numerical values of our heights …………… cm.

b. Mean/average of our heights………………….. cm.

Many a time, the decision taken from the arithmetic mean of some collected data does not agree with the reality. You do not understand the matter, right? Let us then try to understand this through a story.

Suppose some of your friends and all their families decided to go to a picnic. There would be arrangements of different games in the picnic and the winners would be awarded prizes. There would be arrangement of games for the members who are 20 years of age or older. Another game would be for those who are under 20 years. Counting all the family members, you found that there are 9 members who are under 20 years of age. Amongst them, 5 are 3 years old, 2 are 12 years old, 1 is 14 years and 1 is 19 years. So, the average/mean age of these 9 members is:

= (3+3+3+3+3+12+12+14+19)/9

= 72/9 = 8 years

Now suppose a game of quiz has been arranged keeping this mean/average age in mind.

And the questions of this quiz are appropriate for 8 years old students:

a) 27+21+15 =?

b) 2639-305 =?

c) 79 x 63 =?

d) How many Tk 20 notes = Tk 500?

Hope you can understand what the results of this quiz may be.

Children who are 3 years old will be unable to answer these. Again, those who are 12,14 and 19 years old, they will be able to solve them very easily. Hence nobody will have fun in the game. Here, computing the mean/average is correct, but the application is not appropriate. So we can conclude that the idea of real situation cannot always be understood from the idea of mean/average. So, it is essential to know which number/numbers occur in the middle and which numbers occur maximum times when the data is arranged in some order of their values.

Median

Median is the number which occrs in the middle of the collected Data. The value which divides the data into two equal parts when the given data is arranged according to some order of their values is the Median of the data.

Determining the mean/average is not always useful in making decisions on many occasions of our daily life. In such cases the median plays relatively better role. For example: in the game you played in the picnic, you obtained the average age was 8 years. But if you had arranged the ages of the 9 members from the lowest to the highest, i.e., in increasing order, then the numbers would be – 3, 3, 3, 3, 3, 12, 12, 14, 19. Here the person in the middle is 3 years old. This 3 is the median of the numbers. If the quiz or the questions of the game are made suitable for the 3 years old children, then the questions will be relatively better than those made for the average age of 8 years old.

To understand the idea of Median better, observe following examples:

1\. 10 9 12 6 15 7 8 14 13

↓

Arranging the data in order of their values, get

↓

6 7 8 9 10 12 13 14 15

↓

~~6~~ ~~7~~ ~~8~~ ~~9~~ 10 ~~12~~ ~~13~~ ~~14~~ ~~15~~

↓

Median = 10

2\. 6 4 7 8 5 12 10 11 14 15

↓

Arranging the data in order of their values, get

↓

4 5 6 7 8 10 11 12 14 15

↓

~~4~~ ~~5~~ ~~6~~ ~~7~~ 8 10 ~~11~~ ~~12~~ ~~14~~ ~~15~~

↓

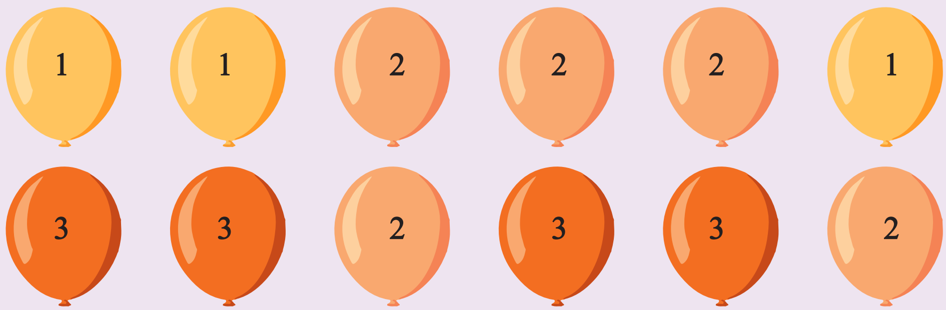
Median = (8 + 10) ÷ 2 = 9

3. Identify the median from the following objects

1 2 3 4 5 6 7

Mode

a. Given a set of data, the data that occur mostly, is called the Mode.
Observe the following example:



b. Among the numbers in the picture above, 1 occurs 3 times, 2 occurs 5 times, 3 occurs 4 times. Since 2 occurs the most, 5 times, 2 is the Mode of the given data.

5 7 6 2 8 3 2 1 4 5

↓

Arranging the numbers in order of their values, get

↓

1 2 2 3 4 5 5 6 7 8

↓

Since 2 and 5 occur the most, 2 times, hence 2 and 5 are Mode

c.

3	4	7	8	5	9	2	6	10	11

Arranging the data in order of their values, we get

2	3	4	5	6	7	8	9	10	11

Since each number in the data occurs once, that is there is no repetitions, hence the data has no Mode.

Line Graph

Line Graph is a presentation of data in a picture form which continuously changes with time. In line graph the data is primarily represented by dots. Then the separate dots are joined by straight lines to form a line graph. Line graph is formed by two axes or lines. One of them is horizontal axis/line and the other is vertical axis/line. The horizontal axis/line is known as the x-axis and the vertical axis/line is known as y-axis. The point, where the x-axis and y-axis intersect each other, is known as the origin. In line graph, the lines are horizontally arranged and change from left to right.

Let us draw a line graph with the following information:
In a game of cricket of the Bangladesh cricket team, runs obtained per over are given in the following table:

Over	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Run	8	10	6	5	0	8	6	4	7	12

Line Graph

In a graph paper, a line graph is drawn by considering every five little squares along the horizontal x-axis as an over and every two little squares along the vertical y-axis as the number of runs.

Assigned Work/Problem: With the help of your guardian, fill up the following list of the expenses for the last 6 months on grocery, education, transport, medical and other miscellaneous things. Prepare a plan, how you can make adjustments on the monthly expenses so that from the next month you can save 10% of the average expenditures of the last 6 months.

Expenditures in my house	January	February	March	April	May	June
Grocery						
Education						
Transport						
Medical						
Miscellaneous						
Total						

Using the completed list, answer the following questions:

a) Find the average grocery expenses from the list.

b) Find the median of the medical expenses of the last six months.

c) In which item the mode occurs in the prepared list?

d) Draw a line graph using the item wise expenses in the list.

At the end of this assignment, your guardians will evaluate your work with comments (Rubrics for the guardians for the evaluation is attached on page 56). Submit your assignment to your teacher together with the comments of the guardians.

Exercises

1. Marks obtained by 40 students in a class test in Mathematics are as follows:

8, 7, 9, 4, 6, 8, 9, 10, 5, 4, 9, 8, 7, 6, 8, 7, 9, 10, 6, 4, 5,
8, 9, 7, 10, 6, 10, 8, 9, 8, 6, 5, 8, 9, 10, 7, 4, 10, 8, 6

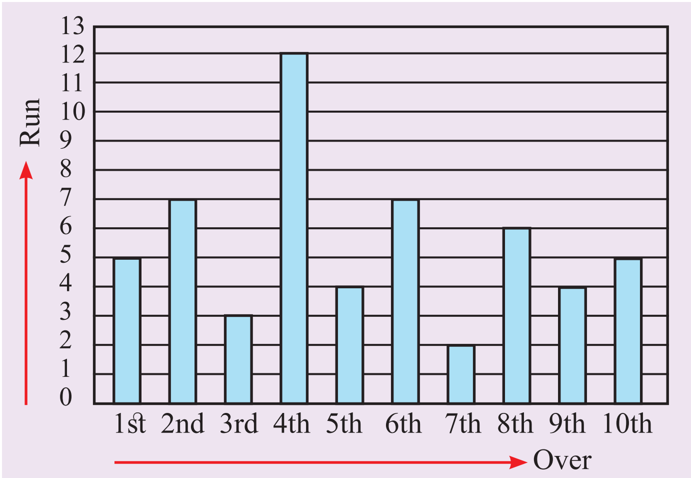
a) Arrange the data in decreasing order.
b) Prepare a table using tally marks.

2. Amiya is a student of class six. The number of students in her school from class one to class six are:

Class	One	Two	Three	Four	Five	Six
Number of students	180	160	150	170	190	200

Draw a bar chart taking the number of students along the vertical line. [Hint: Mark the numbers of students along the vertical line in such a way that all the numbers are in the bar chart.]

3. In a one-day cricket match between Bangladesh and Australia, a bowler of Bangladesh team bowled ten overs. Runs conceded by him in different overs are shown in the bar chart below:



Answer the following questions from the diagram:

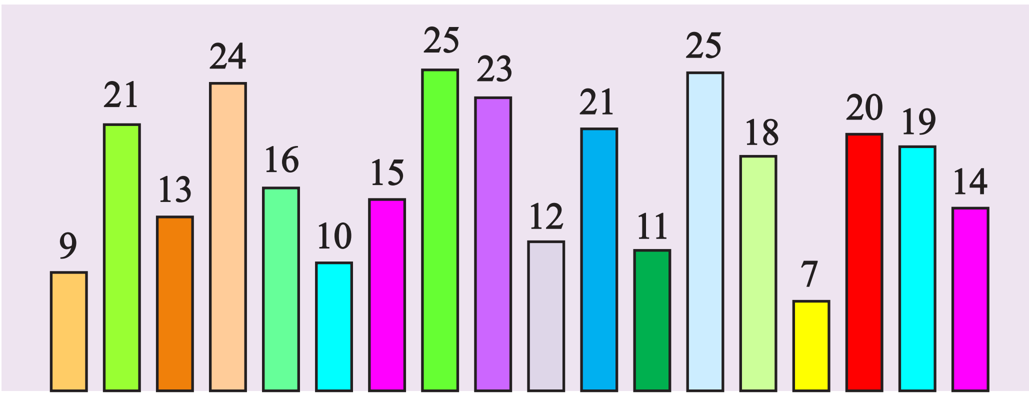
a. In which over the maximum runs were conceded?

b. What is the total run conceded in ten overs?

c. What is the average/mean runs per over?

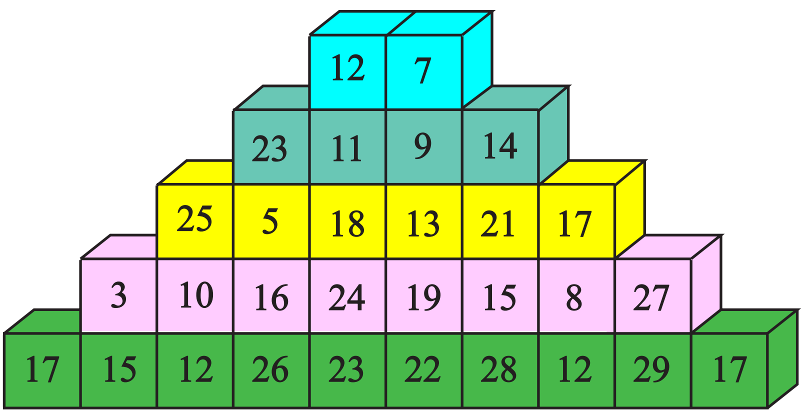
4. Write down the prime numbers less than 50. Find the average/mean and the median of the numbers.

5.



Heights (in metre) of the bars are given. Find the median of the data.

6. Find the average/mean, median and the mode of the data:



7. Talk to 20/25 students of your class/ your previous class/ your following class. Collect the following data (their ages, daily study times, daily games times, daily sleeping times etc) and prepare a list or a table according to the sample below.

Serial Number	Name of student	Age (years)	Daily study (hours)	Daily games (hours)	Daily TV watching (hours)	Daily sleeping (hours)
1						
2						
3						
4						

Using the List or table, find the answers to the following questions.

a) Using the different types of information of the students mentioned in the list, find the mean/average, median and mode of any three. In this case, which one is more effective according to you – comment with justification.

b) Draw a line graph using the daily study time of the students.

c) "Those who have more study time, they have less sleep time"—verify the validity of the proverb from the list prepared by information obtained by you.

d) Is there a relationship between more study time of students with their games time and TV watching time? Find out.

e) Is there any relationship between students having more games time, with their study time, sleeping time and TV watching time? Find out.

f) Write a summary of your own opinion about the differences/ similarities of the study time and games time of the students of the classes of which you collected and analysed the data.

A sample Rubrics to evaluate the ability to analyze more than one result and make logical decisions

The rubrics will be used by the guardians to evaluate the report of their children and the students will submit a copy of this evaluation with the report to their teachers.

Guardians of students will make observations of the report prepared and express their own opinions beside the statements below.		
Field of evaluation	Agree	Disagree
Could compute the mean/average expense of different expenditures of the family		
Could identify the items on which expense is the most		
Through discussion with guardians, coordinating the various monthly expenses of items, made a logical savings plan from the next month, to make 10% savings of the average expenses of last 6 months.		
My child learned the importance of planned expenditures through this work.		
My child became interested in savings by preparing this report through this work		
Overall comments of guardian:		

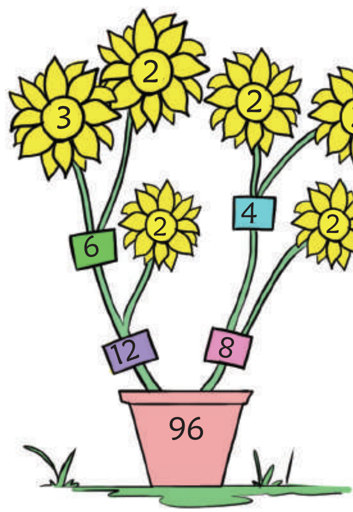
Trees of Prime Factors

In nature, some trees are seen to have no brunches and stalks like Betel Nut tree, Palm tree, Coconut tree and Date Palm tree etc. Again, there are few trees having lots of brunches and stalks like Mango tree, Java Plum tree, and Pepper Plant etc.

Palm tree, Betel Nut tree, Coconut tree and Date Palm tree can be named as Prime trees whereas Mango tree, Java Plum tree and Pepper plant can be named as Compound trees.

You may wonder, what can be the relationship between Prime Factors and Trees!

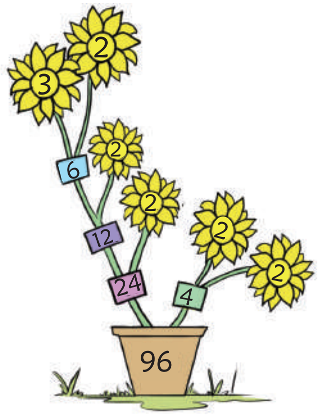
You'll understand it from the picture below.



You'll see that the prime numbers are drawn as yellow flowers. Now, think why we didn't write them like: 3 = 3 or 1 or 2 = 2 or 1

1. Do you know whether 1 is a prime number or not?

Again, a tree of factorization for number 96 can be drawn like below:



Now, you along with your classmates, find out how many more factorization trees can be drawn for the number 96.

At this point, each of you will pick a natural number through lottery.

Find out how many more factorization trees can be drawn for the number you got through the lottery.

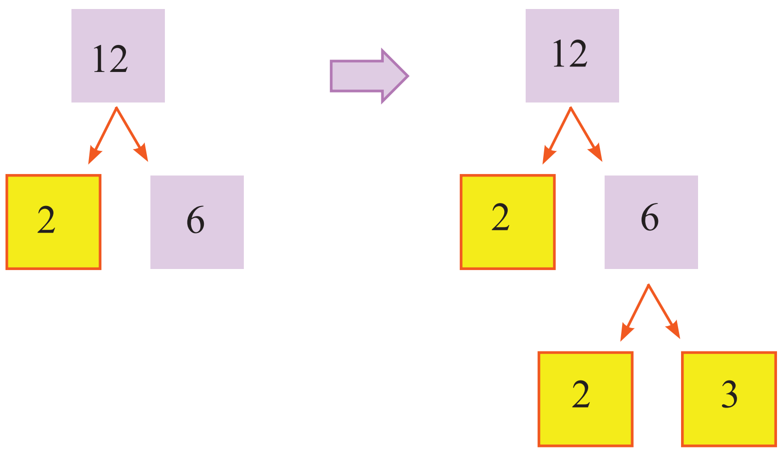
Draw all the factorization trees on a poster paper or on an old calendar and show it to your teachers, classmates and others.

You can draw the trees as you like. Just write the prime numbers in yellow.

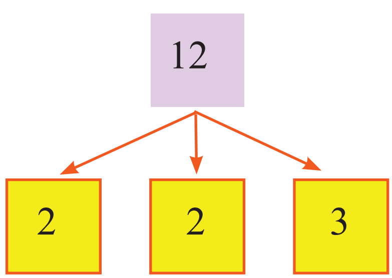
Combining all yours' factorization trees in one place, you can form factorization orchard and then can exhibit to others.

Now, make factorization trees with the following numbers.

However, if necessary, you can draw the tree of factorization from top to bottom as in the picture below. In that case, can you mention the advantages you may have? These tree-like diagrams are called Tree diagram.

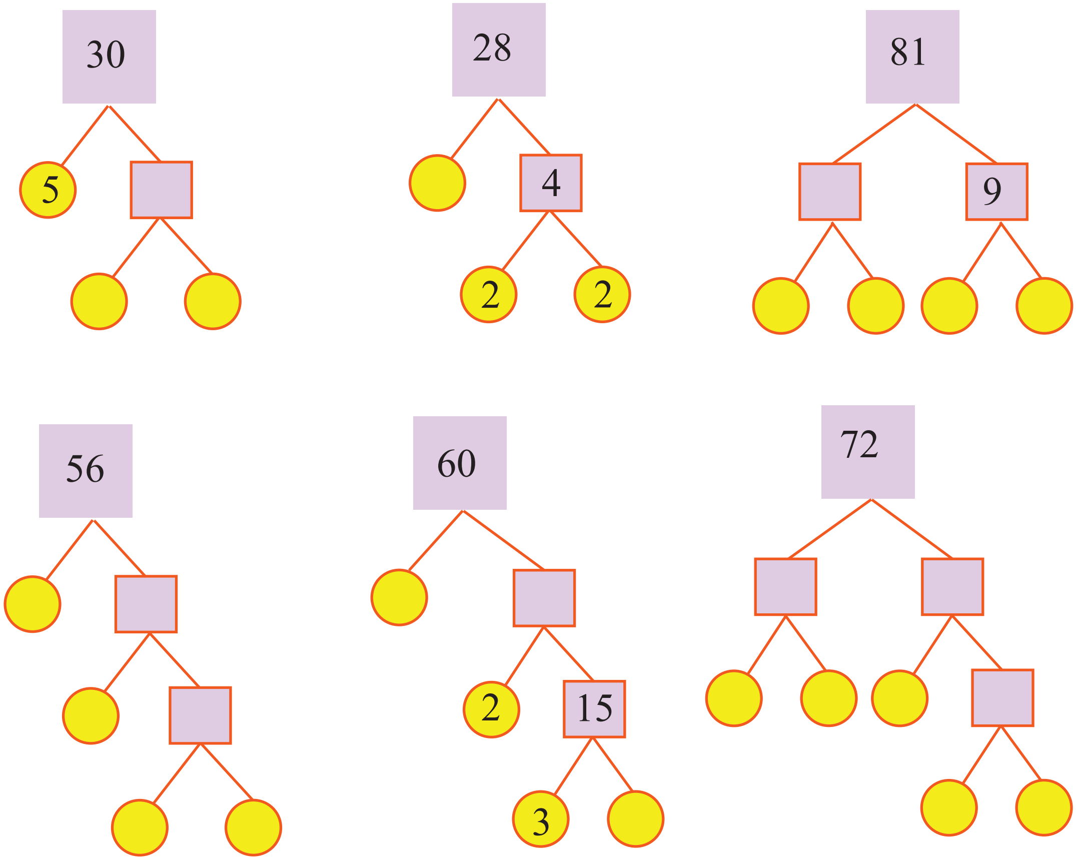


Now, let's see the factorization tree for number 12 given in the picture below.



See, only the prime numbers are taken here from the factorization tree of number 12.

Now, fill up the Trees of prime factors.



The game of Multiple and Factor

Now, we will play a fun game with the multiple and factor of any number.

You surely know how to find out the multiple and factor of a number, right?

Let me share an interesting matter.

Factorization and factor are no different things.

That means, you can use the tree of factorization concept for finding out the factors of any number. 1

Rules of the game:

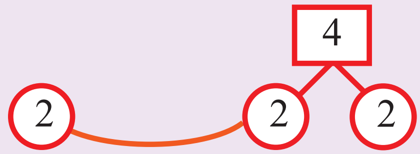
- At first, draw the trees of prime factors of 1st and 2nd number.
- If all the prime factors of 1st number remains present in the 2nd number then:
- The 1st number is the factor of 2nd number and the 2nd number is the multiple of the 1st number.
- Again, if all the prime factors of 2nd number remains present in the 1st number then:
- The 2nd number is the factor of 1st number and the 1st number is the multiple of the 2nd number.

Look at the pictures below for better understanding:

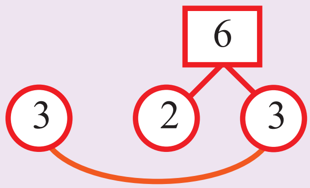
2nd number is multiple of 1st number
1st number is multiple of 2nd number

2nd number is multiple of 1st number
1st number is multiple of 2nd number

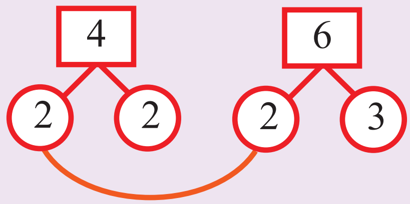
2nd number is not multiple of 1st number
1st number is not multiple of 2nd number



2nd number is multiple of 1st number
1st number is factor of 2nd number
but not multiple



2nd number is multiple of 1st number
1st number is factor of 2nd number
but not multiple



2nd number is not multiple of 1st number
1st number is not multiple of 2nd number

- **Then fill up the table below with √ or × sign.**

1st Number	2nd Number	Is the 1st number a factor of the 2nd number?	Is the 2nd number a multiple of the 1st number?	Is the 2nd number a factor of the 1st number?	Is the 1st number a multiple of the 2nd number?
2	2	√		√	√
3	3				
2	3				
2	4	√	√	×	×
3	6				
4	6				

Pair work:

- Select two numbers for each pair through lottery.
- Play the game of multiple and factor in each pair, taking the two numbers in the lottery as the 1st and 2nd numbers.

The game of H.C.F

You have learned multiple methods of determining H.C.F in your previous class. You're also familiar with the system shown below:

$$\begin{array}{r|l} 2 & 18,\ 12 \\ \hline 3 & 9,\ 6 \\ \hline & 3,\ 2 \end{array}$$

H.C.F = $2 \times 3 = 6$

But, can you tell why H.C.F is determined like this?

Let's understand the meaning of H.C.F.

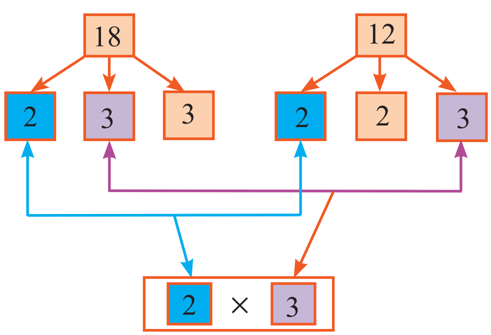
H.C.F stands for Highest Common Factor.

You have already come to know from 'The game of Multiple and Factor' that:
If all the prime factors of a number remain present in another number then the ١st number will be a multiple of the ٢nd number.
The common factor of two numbers will be such a number whose all prime factors will remain present in those two numbers' tree of prime factors.
Now, H.C.F means Highest Common Factor that is the biggest common factor.
In that case, if you can find out all the prime factors of two numbers from their tree of prime factors then the multiplication of those prime factors is actually their H.C.F.
If you want, you can try and see if any number greater than this multiplication can be a common factor for both the numbers or not?

Now, think how you may play the game of determining H.C.F with help of tree of prime factors?
Rules of H.C.F's Game:

- Draw trees of prime factors for two numbers.
- Identify the prime factors that are common in both the tress. These are the common prime factors of those two numbers.
- Now, the multiplication of common prime factors will be the H.C.F of that number.

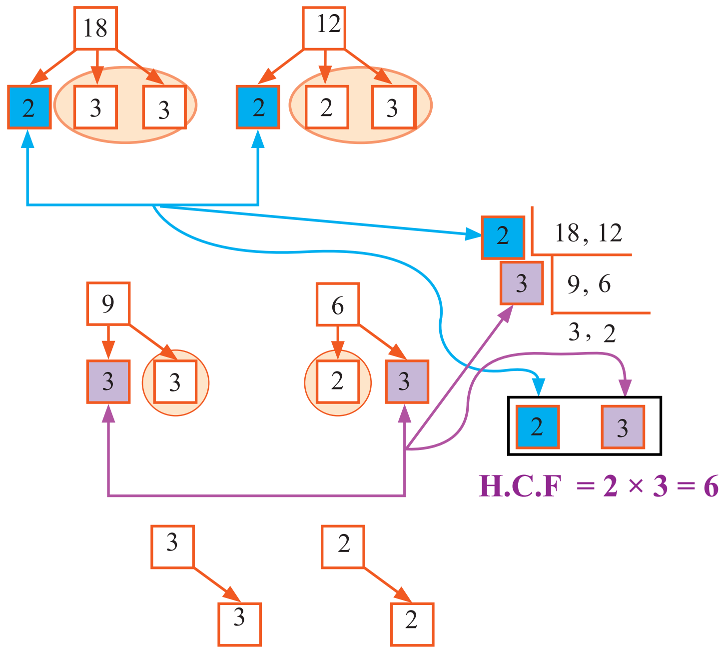
Please see how the H.C.F of 18 & 12 is determined through the game of H.C.F in the picture below:



H.C.F = $2 \times 3 = 6$

Can you find any similarity between the method of determining H.C.F with the tree of prime factors as shown above and the method shown at the beginning of the H.C.F's game section?

If you see the picture below, you'll easily understand that both the systems are identical.



Even though 1 is not there in the tree of prime factors, yet it is the factor for all numbers.

If two numbers do not have any common factor other than 1, that is, if their H.C.F is 1, then we call the two numbers Co-prime number.

For example: H.C.F of 4 & 9 is 1. So, 4 & 9 are Co-prime numbers.

Individual Work : Everyone must select three numbers of two digits. Then, determine the H.C.F of the three numbers through the game of H.C.F and with the help of the tree of prime factors.

Let's Do the following task

Now, determine the H.C.F of the following numbers through the game of H.C.F and with the help of the tree of prime factors

1) 28, 24
2) 35, 25, 105
3) 45, 18, 99
4) 28, 48, 72
5) 31, 32, 341

Determine the HCF of the numbers making a list of factors.

Determining HCF through the division method following Euclidean method

Determining H.C.F in Picture

You already have known two systems of determining H.C.F for two numbers.

- **First System:**
  - Make a list of all the factors of those two numbers.
  - Find out the common factors of those two numbers from the list.
  - Among the common factors, the highest number found will be the H.C.F of those two numbers.

Example

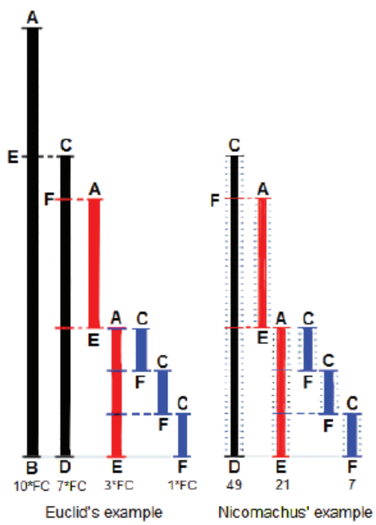
Factors of 20	**1, 2, 4,** 5, 10, 20
Factors of 32	**1, 2, 4,** 8, 16, 32

HCF = 4

- **Second System:**
  - Factorize the two numbers with the help of tree of prime factors.
  - Find out the common prime factors of both the numbers.
  - The multiplication of all common prime factors will be the H.C.F of those two numbers.

In both the systems mentioned above, for making a list of factors or even for doing prime factorization, you need to divide both the numbers many times. And if the numbers are too big then it would take much time to determine the H.C.F following the above methods.

For making the task of determining H.C.F a little easier, mathematician Euclid (300 BC) found a different but interesting system. However, Nichomacus, another mathematician also knew of this method. Check the image beside.



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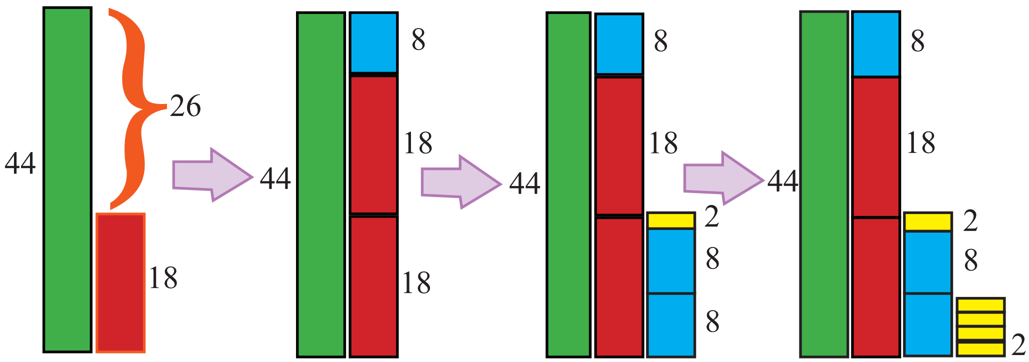
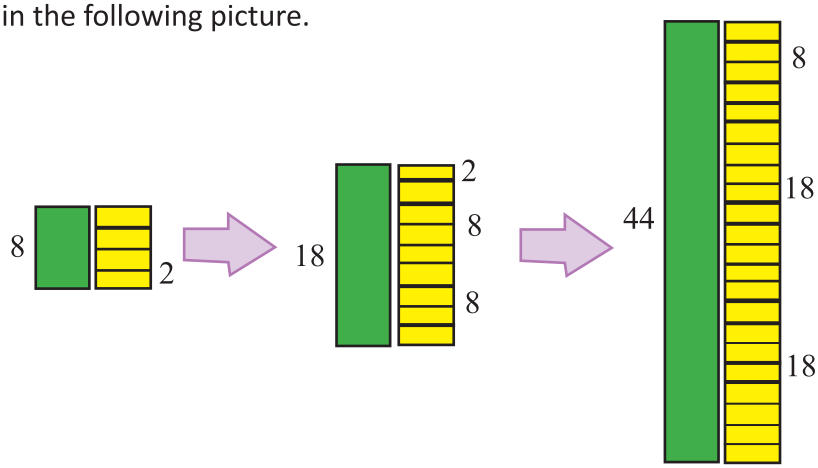
Now, the H.C.F of 44 and 18 will be determined following that system.



- At the very outset, cut a strip of paper that is 44cm long and 5cm wide using a scale.
- Now, cut a few strips of paper that is 18 cm long and 5cm wide. (Here, the measurement of length is important in determining the H.C.F. So, you can take any width other than 5 cm as per your convenience. However, it would be convenient if all the widths are same
- Now, place the 18cm strip beside the 44 cm strip. It's yet 26 cm to be equal to 44 cm.
- Determine the maximum number of strips of 18 cm length that can be placed without exceeding 44 cm length?
- In the picture you can see that, after placing two 18 cm strips only 8 cm is left.
- Now, make a few 8 cm long strips and place those beside one 18 cm long strip.
- In the picture you can see that, after placing two 8 cm long strips only 2 cm is left to meet the length of 18 cm.
- Now, make a few 2 cm long strips and place those beside one 8 cm long strip.
- See in the picture, after placing four 2 cm long strips, the required 8cm length is met.
- Now, our task is finished. At last, we could fill up the 8 cm length of the strip with 2cm long strips. Therefore, the H.C.F of 44 and 18 will be 2.

But we need to know how we could determine the H.C.F through filling up paper strips. The answer lies in the concept of multiple.

See in the following picture.



In the end, we could fill up one 8cm long strip with 2 cm long strips.

So, 2 is the factor of 8.

From the picture, it is also evident that 2 is the factor of both 18 and 44.

Therefore 2 is the doubtless common factor of 18 and 44.

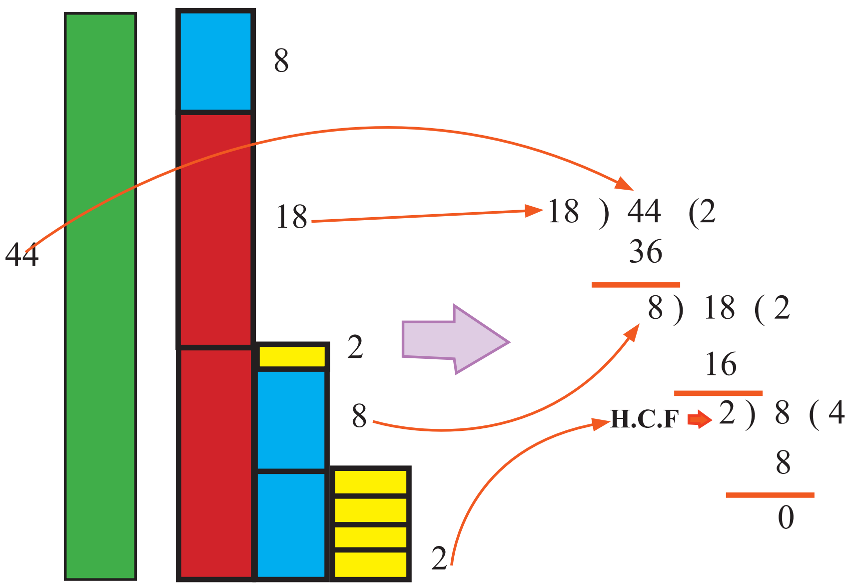
Now, the last question to you is:

Can it be proved from the picture, following the above method, that 2 is the biggest or Highest Common Factor of 44 and 18?

Think individually first.

Then share your thoughts or opinions to everyone according to the teacher's instructions and find out the proof through group discussions and other needful activities.

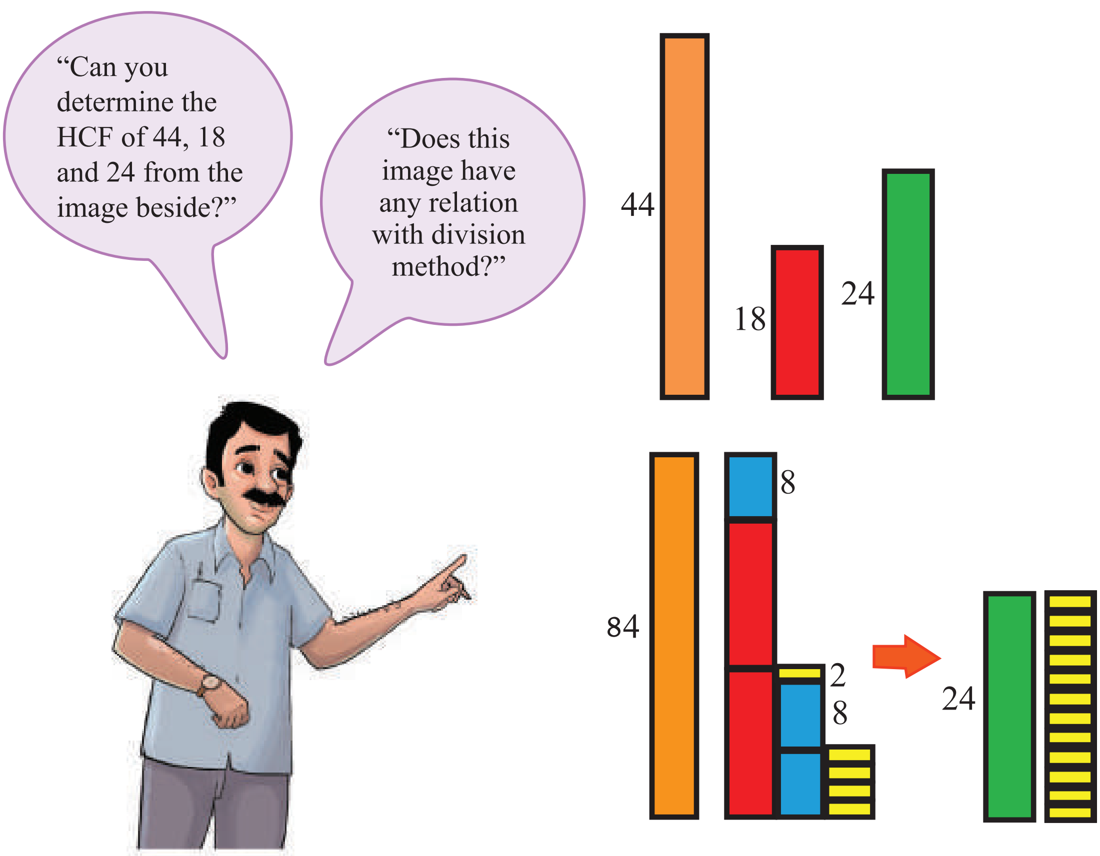
The relationship between, division process and the activity for determining H.C.F as per Euclidean method:



Individual Task:

- Now, each of you must select two numbers through lottery.
- Make paper blocks with the numbers you selected through lottery and then complete an activity of determining H.C.F. Besides, show by drawing, the relationship between determining H.C.F through the method of division with the two numbers selected through lottery.
- Draw all your work on poster paper / old calendar and present it to your teachers and classmates in the next class by attaching blocks of paper with glue.

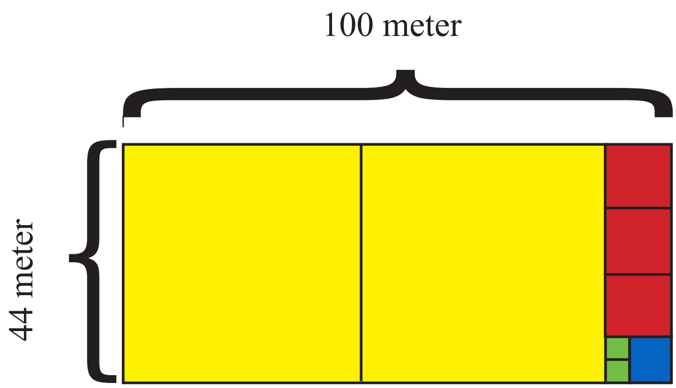
Determining the H.C.F of three numbers in picture:



Exercise

1) Through the picture, determine the H.C.F of the following numbers according to the method of division.
   (a) 24, 45, 62 (b) 56, 78, 90 (c) 120, 56, 78 (d) 99, 33, 123 (e) 95, 57, 23

2) The H.C.F of 100 and 44 can be determined from the following picture. Can you tell how?



(Inside the rectangle in the picture, there are many squares of different colours)

Let's take a look at some examples of how and why H.C.F is needed in various real life problems.

3) There are two ropes of 15 m and 40 m length. Cut these two ropes into small pieces of the same length so that no part of the rope is damaged. What can be the maximum length of the small pieces?

4) A shopkeeper sells candles in both packets of 12 and 8. To have one candle for each candle stand, what is the minimum number of candles and candle stands that Ayesha has to buy?

5) A florist wants to arrange 24 bouquets in different rows. In how many different ways can he arrange them with the same number of bouquets in each row?

6) 210 Oranges, 252 Apples and 294 Pears are evenly packed in cartons so that no fruit is left out. What is the maximum number of cartons that will be needed there?

7) The length, width, and height of a room are 6 m 80 cm, 5 m 10 cm and 3 m 40 cm respectively. You will be given a stick only, not a scale. The length of that stick will be as you want but you can only demand it once. That means you'll get only one stick. With this stick you have to make sure that the length, width and height of the room are measured accurately. What is the maximum length of stick you can ask for?

8) H.C.F of two numbers is 6, if one number is 42, find the other number?

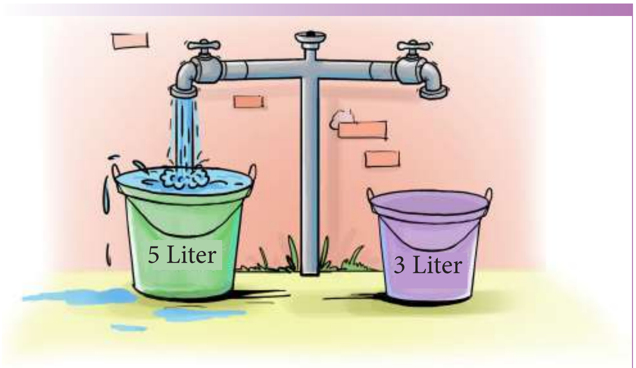
9) Activity with the help of bucket and water:

   a) How to measure 4 liters of water with 3 liter and 5 liter water buckets?

      In this case, there will be no measuring marks on the bucket. Again, other measuring instrument such as scale or measuring scales etc. cannot be used.

   b) Which of the following amount of water can be measured with 4 liter and 6 liter water buckets?

(In this case, there will be an opportunity to keep in other containers for 8, 8, 9, 10 liters)

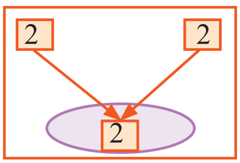


Amount of water (Liters)	Can it be measured with 4 liter and 6 liter water buckets?	Write how to measure step by step
1		
2		
3		
4	√	
5		
6	√	
7		
8		
9		
10		

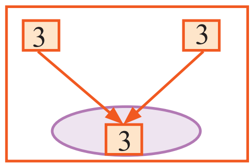
Individual Task : Each student will find a similar real life problem and solve it. They will also present it in the next class.

Game of L.C.M

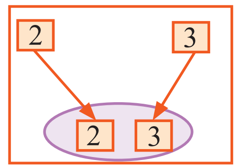
1. Take two numbers and then ask the students to draw the tree of prime factors of those two numbers. Also, ask the students to draw a separate L.C.M box below the tree where there will be the prime factors of L.C.M. From their previous idea, the students will take the factor from the 1st number and for determining its multiple, they'll match the factor of the 1st number with the same factor from the 2nd number, if there is any. Moreover, they'll look for more prime factors of the 2nd number and if found then they'll write down those prime factors in L.C.M box to determine the multiple of 1st number. The teacher will demonstrate it to the students first.



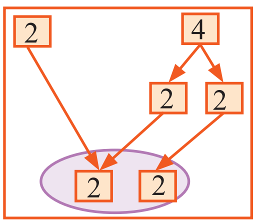
L.C.M = 2



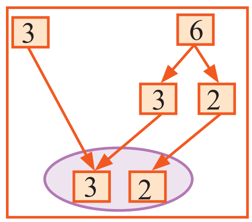
L.C.M = 3



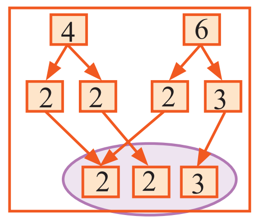
L.C.M = 2 × 3 = 6



L.C.M = 2 × 2 = 4

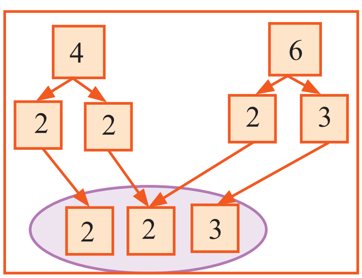


L.C.M = 3 × 2 = 6

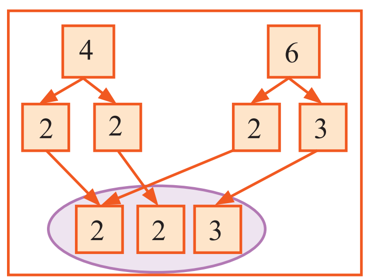


L.C.M = 2×2×3 = 12

- Here, drag down the factors of the 1st number as shown below. And for the 2nd number, if there is any same factor then that has to be matched with the 1st number. If there is more left, drag it down to make a multiple of the 1stnumber.Here, from the 1st number 4, at first 2, 2 prime factors will come down. Then match the same prime factor 2 from the 2nd number 6 and drag 3 down.



L.C.M = 2×2×3 = 12

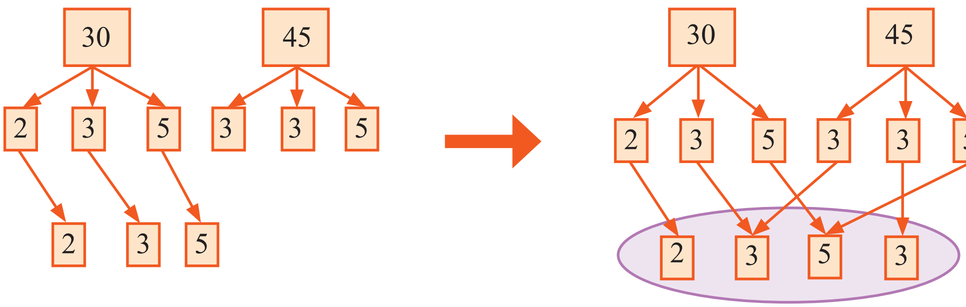


L.C.M = 2×2×3 = 12

2. If you can find L.C.M following this method, then take two big numbers. Let's suppose the numbers are 30 and 45. At first, then you will create tree of prime factors for these two numbers.

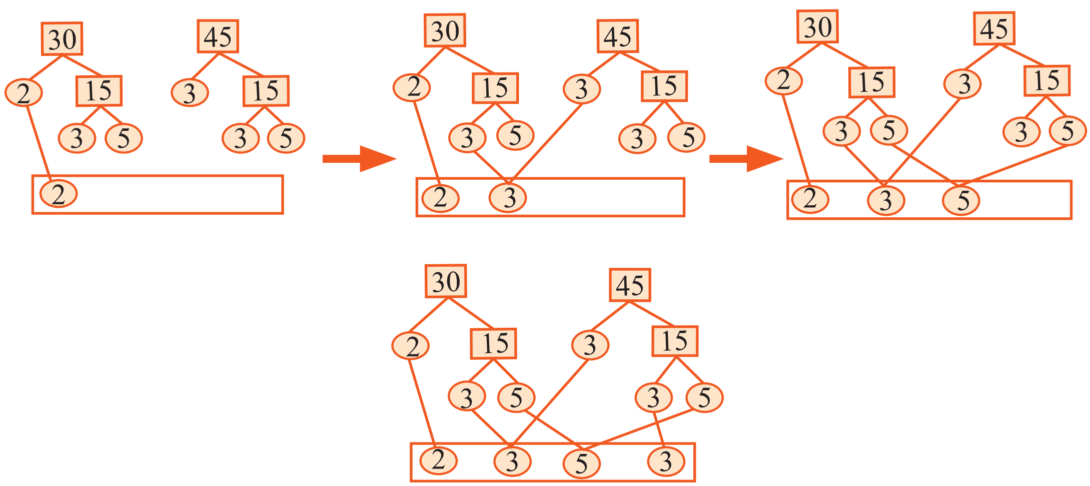
Then, according to the rules described earlier, you'll drag down the prime factors of the 1st number in L.C.M box. After that, they'll match the similar prime factors with the 2nd number and drag down the rest of the prime factors.

Finally, you'll find out the L.C.M from the prime factors of L.C.M itself.



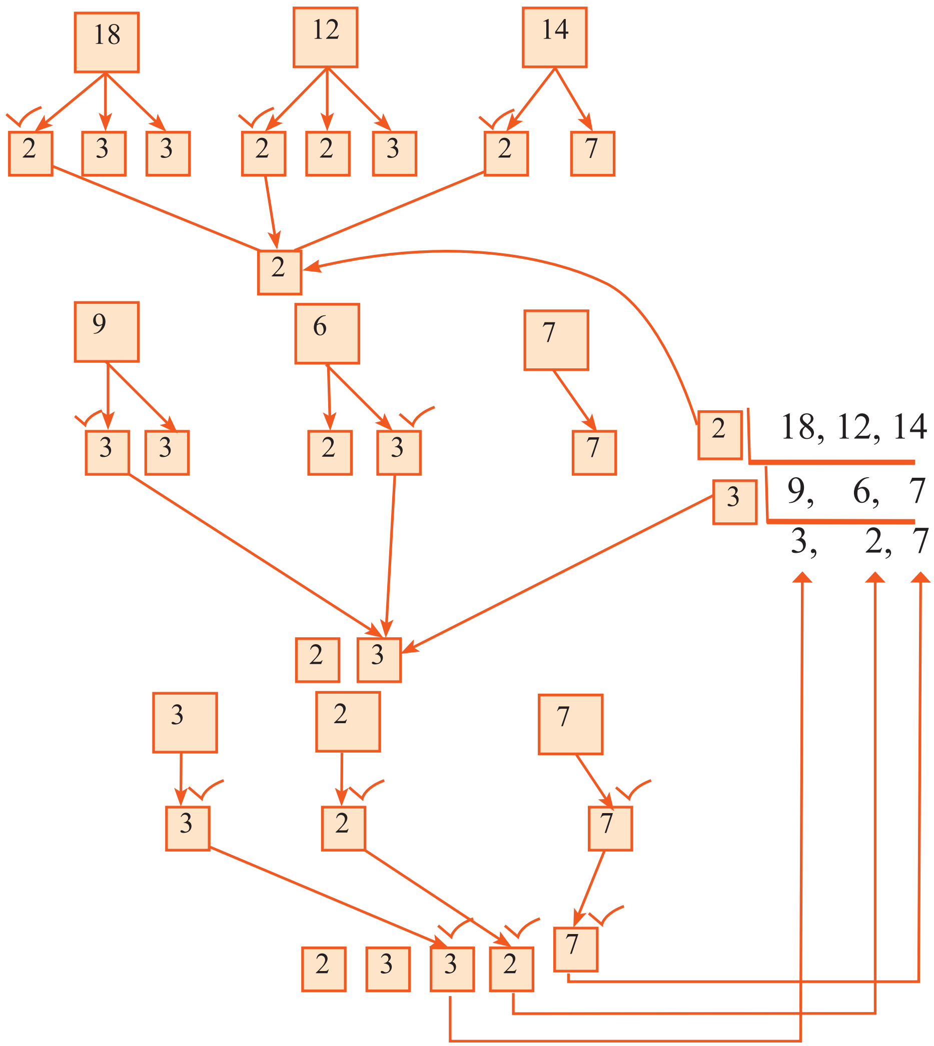
L.C.M $= 2 \times 3 \times 5 \times 3 = 90$

- In this way of finding out L.C.M, you will be able to establish the relationship between finding out L.C.M through finding out the common multiples and finding out L.C.M from the prime factors. In the same way, if you consider the prime factors of L.C.M in both the 1st and 2nd number and if you find any similarity then you need to drag it down in the box below using an arrow sign. At the same time, the dissimilar ones have to be dragged down as well following the same manner. If you find the L.C.M in this method then you'll get the same L.C.M.



L.C.M = $\boxed{2 \times 3 \times 5 \times 3}$ = 90

- This time you will also get an idea of how to find out the L.C.M of 18, 12, 14.



The above mentioned procress of measuring L.C.M is called Euclidean Procress.

Try to answer the following questions

a) In this process, why this minimum two numbers have been chosen or divide with this number?

b) At first, Instead of 2, divide with the numbers 7 or 3 and see whether the L.C.M becomes same

What we know measuring L.C.M through Euclidean Procress.

Determining the L.C.M of 12, 18, 20 and 105.

$$
\begin{array}{r|l}
2 & 12,\ 18,\ 20,\ 105 \\ \hline
2 & 6,\ 9,\ 10,\ 105 \\ \hline
3 & 3,\ 9,\ 5,\ 105 \\ \hline
5 & 1,\ 3,\ 5,\ 35 \\ \hline
 & 1,\ 3,\ 1,\ 7
\end{array}
$$

Determined L.C.M= $2 \times 2 \times 3 \times 5 \times 3 \times 7$

Let's see the rule from the given example:

- Line (L) is drawn below the numbers by writing the numbers in a row with the (,) sign.
- At least two of the given numbers are divided by common prime factors.
- Quotient of the numbers that are infinitely divisible by the factor is also written below.
- Those not divisible are written unchanged.
- The numbers that are in the bottom row were worked out following the previous rule.
- Dividing this way, when the numbers in the bottom row became co-prime numbers then they were not divided any longer.
- The consecutive product of the numbers in the bottom row and the divisors is the determined L.C.M.

The maximum number of prime factors of L.C.M is shown in the book. Explain that method following the above method.

Least Common Multiples (L.C.M)

We know, The multiples of 4: 4, 8, 12, 16, 20, $\boxed{24}$, 28, 32, 36, 40, 44, $\boxed{48}$

The multiples of 6: 6, 12, 18, $\boxed{24}$, 30, 36, 42, $\boxed{48}$, 54 etc.

And , The multiples of 8: 8, 16, $\boxed{24}$, 32, 40, $\boxed{48}$, 56, 64 etc.

It is seen that, the common multiples of 4, 6 and 8 are 24, 48 etc. and the least multiple is 24.

$\therefore$ the L.C.M of 4,6 & 8 is 24.

$4 = 2 \times 2$, $6 = 2 \times 3$, $8 = 2 \times 2 \times 2$

Here, 2 appears for a maximum of 3 times and 3 appears for a maximum of 1 time among the prime factors of the numbers 4, 6, 8.

So, if a consecutive multiplication is done taking 2 three times and 3 for a single time then we get, $2 \times 2 \times 2 \times 3$ or 24, which is the L.C.M of the given numbers.

- The highest numbers of L.C.M's prime factors are already shown in the book. Explain that system through the above process and show reasons.

Individual Task :

Now, each of you please select two numbers through lottery.

Make paper blocks with those numbers and take help from the tree of prime factors to determine the L.C.M following all the methods as discussed in the 'Tree of L.C.M'.

Draw all your work on poster paper /old calendar and present it to your teachers and classmates in the next class by attaching blocks of paper with glue.

Exercise:

1) Determine L.C.M following all the possible ways as discussed in the section 'Tree of L.C.M' with the help of tree of prime factors.
(a) 14,15,12 (b) 66,78,100 (c) 120,56,60 (d) 55,15,143 (e) 25,57,95

2) Relation between L.C.M & H.C.F
The prime factors are being determined taking two random numbers 10 and 30.
$10 = 2 \times 5$, $30 = 2 \times 3 \times 5$
H.C.F of 10 and 30 = $2 \times 5 = 10$
And, L.C.M = $2 \;\; 3 \; 5 = 30$

Again, the product of the numbers 10 and 30 = $10 \times 30 = (2\times5) \times (2\times3\times5)$
= H.C.F × L.C.M

∴ The product of two numbers is equal to the product of H.C.F and L.C.M.
Product of two numbers= H.C.F of both the numbers × L.C.M of both the numbers
Now,
'The product of two numbers is equal to the product of both the numbers' H.C.F and L.C.M.'

Will you be able to prove the above mathematical statement for any two numbers through the method as discussed in the 'Game of H.C.F' and 'Tree of L.C.M' sections?

Let's take a look at some examples of how and why L.C.M is needed for various real life problems.

3) What is the minimum number of students that can be arranged in groups of 3, 4, 6 and 8 so that no one is left out?

4) There are 2 types of buses in a local bus service that starts from 8 am. The first type of buses leave after every 15 minutes and the second type of buses leave after every 20 minutes. How many times do the first and second type of buses leave at the same time between 8 am and 11 am on a given day?

5) Three painters, Ron, Habib and Shelley, are designing a hotel room. The hotel has room numbers from 15 to 200. Ron has to work in all the rooms. Habib has to work in the rooms where the room number is a multiple of 3. Shelley has to work in the rooms where the room number is a multiple of 5. In which rooms will they all work together?

6) Sara goes to a shopping mall every 6th day in a week. Andy goes to the same shopping mall every 7th day. How many times will they meet each other in the mall in December and January if the counting starts from 1st December?

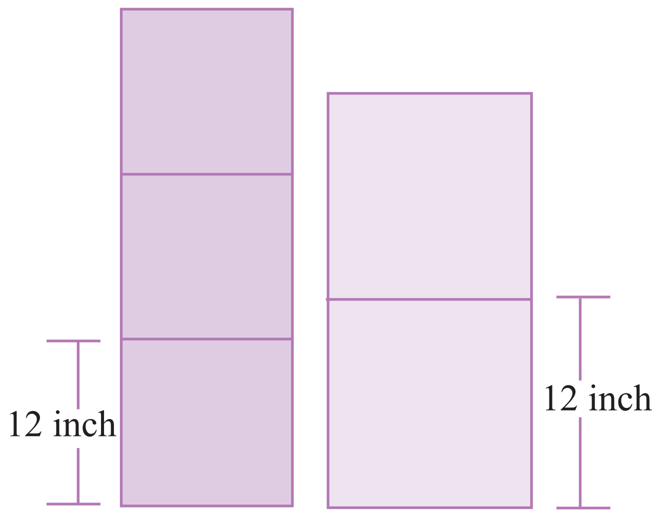
7) Sami can jump 4 steps at a time and Nina can jump 5 steps at a time. If the two starts jumping together, at what step will they meet?

8) Amia has a music class every 2nd day and a painting class every 3rd day. On which day will she have both the classes?

9) Today, both the football team and the basketball teams were playing. The football team plays 3 days in a week and the basketball team plays 5 days in a week. When next the two teams will play on the same day?

10) You look at your friend in every 4 seconds and smile and your friend looks at you in every 8 seconds and smiles back. When you and your friend will laugh at the same time? (Hint: Smile among yourselves and find out)

11) In the picture two separate piles are being made side by side using two different shaped square boxes. What is the minimum number of orange and blue boxes that will be required to equalize the height of the two piles? What is the minimum height required for the two piles to be equal?



12) In a marathon race, two people start drinking water after starting the race at regular intervals. The first person drinks water in every 9 minutes. 72 minutes after the start of the race, two men drank water at the same time. At what interval does the second person drink water? How many times does the second person drink water in 72 minutes?

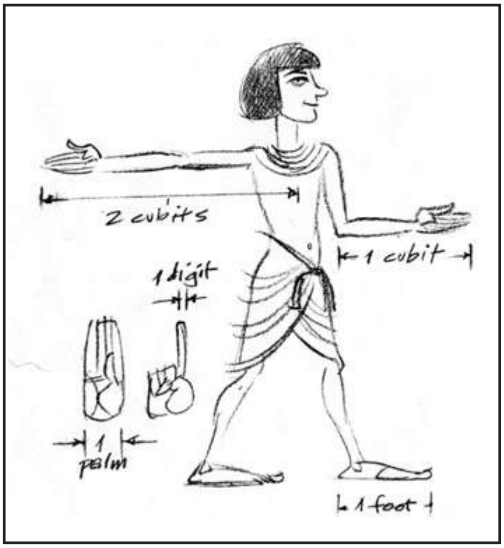
13) Bus A and Bus B are two intercity service of Dhaka. Bus A service leaves the bus stand in every 60 minutes and Bus B service leaves the same bus stand in every 80 minutes. Everyday they start their journey at 6 AM. How many times and at what times in a day they leave the bus stand together?

Individual Task : Each student will find a similar real life problem and will present it to the next class with solution.

Measurement of Length

We have to the task of measuring in almost every task of our daily life. When you go to market for buying a variety of essential commodities such as: rice, pulses, oil, salt, sugar, rope, electricity, etc., then the shopkeeper measures the commodities according to your needs. And we call this measuring system as measurement. You must have seen that, the shopkeeper doesn't measure all the commodities in the same system. For example: The tool he uses for measuring rice and pulses is different from the tool he uses for measuring rope and wire. We measure different things at different times by comparing all these ideal quantities. And this ideal quantity is known as unit.

You must be wondering to know, why this measurement system is needed. Just think, if the system of measurement were not invented, could a tailor made your clothes to the exact size of your body? Again, you can easily tell the distance between your school and home just because of this measurement system. You'll have more fun in knowing that from the very beginning of human society, humans have created a variety of measurement systems. The earliest traces of the measurement system are found through the inhabitants of ancient Egypt, Mesopotamia, the Indus Valley, and from the aboriginals of Ilam (located in Iran). The earliest unit of length measurement came from 'cubit', a conventional unit commonly used by the then Egyptians. They used 'cubit' just the way we use 'meter' now.



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If you look at the picture minutely then you will understand how the Egyptians measured length by using their hands. The length of a normal cubit was from the elbow to the tip of the middle finger. It was further divided into "Bighot", the distance between the thumb and the little finger (half a cubit).

Again, time was measured by the periodical span / duration of sun, moon, and other heavenly objects. And if it was necessary to measure the carrying capacity of earthen or metal vessels then those vessels were filled up with seeds and grains, in this way both their carrying capacity and volume were measured. After the system of weighing was invented, grain and stone weights were considered as the ideal. The gold sold by the gold sellers has 18 carats, 21 carats or 24 carats inscribed/engraved/written in them. You'll be surprised to know that, the carat unit, used to measure gold, has its origin from caraway seeds. But the problem is, not all human hands are of the same size; again, not all grain seeds are of the same size. For all these reasons, people felt that it was necessary to determine a standard or specific measure for doing any measurement.

You'll understand this better in the upcoming pages of this chapter. Now, let's do the work given below.

Observe your classroom minutely. Now write the name and approximate size of the classroom including its doors, windows, benches for keeping your books and notebooks, table, black or white board etc. and fill up the table below. You can do the same task for your reading room as well. This task will be easier for you if you have idea about feet, meters and inches. If necessary, discuss the matter with your teacher or your father/ mother/elder siblings.

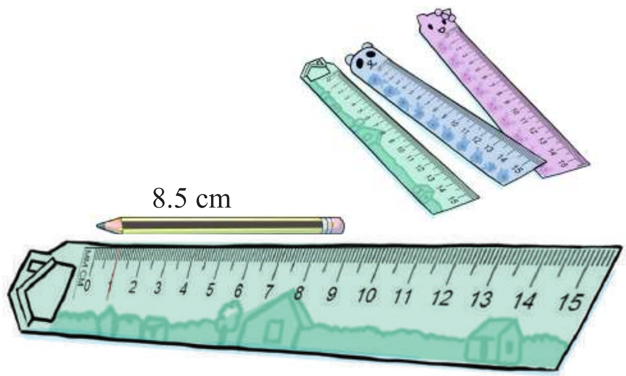
Measurement of length					
Serial number	Name of the measured thing	Approximate Measurement/Size			
		Hand	feet	meter	Inch
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Highlight the instruments you have for determining the approximate measurement of the table accurately. Write down the names of the instruments and describe their units to one of your peers.

Let's Make Paper Scales

Materials: Old calendar or hard paper, Glue, Centimeter Scale, Pencil, Scissors Method:

1. Cut two paper pieces of same size from the old calendar or hard paper considering the length and width of centimeter or inch scale.
2. Attach the two paper pieces together with glue. This will make the body of the scale harder.
3. Now, cut two pieces of white paper and attach them in both the sides of the scale with glue.
4. Place a centimeter or inch scale on any side of the scale, and mark the lines aligning to the lines of centimeter or inch scale with different colour pens.
5. Mark the centimetre lines longer and with different colour. Also, place digits 0, 1, 2, 3, 4, 5, 6…… in the beginning point of the line.
6. Now, design the scale as per your own likings and it is made.



Draw the table in your exercise book. Now fill in the table by measuring the approximate length of pencil, pen and eraser with the centimeter scale you have made with paper. Then again, measure the accurate length of the above items with a purchased scale and write it in the table. Now, write comments comparing the lengths you got by measuring in two different ways.

Serial number	Name of the measured thing	Approximate length in centimeter (with the scale you made)	Exact length in centimeter (with the purchased scale)	Comment on the comparison of the measurement
1				
2				
3				
4				

Joya went to visit her maternal uncle's house with her parents during school holiday. Joya has a very good relation with her cousin Anik as they both read in class six. They usually gossip about their schools whenever they get time. They also discuss about their studies. Joya wants to know how far Anik's school is from home. Anik thought for a while and said it's about 3 kilometers. Joya imagined the distance of 3 kilometers. Anik sometimes goes to market with his father for purchasing necessary goods. One day he went to a store to buy electric wire. The shopkeeper asked, how many yards do they need? Once Anik's father told the amount, the shopkeeper measured it by the scale drawn on his table and gave it.

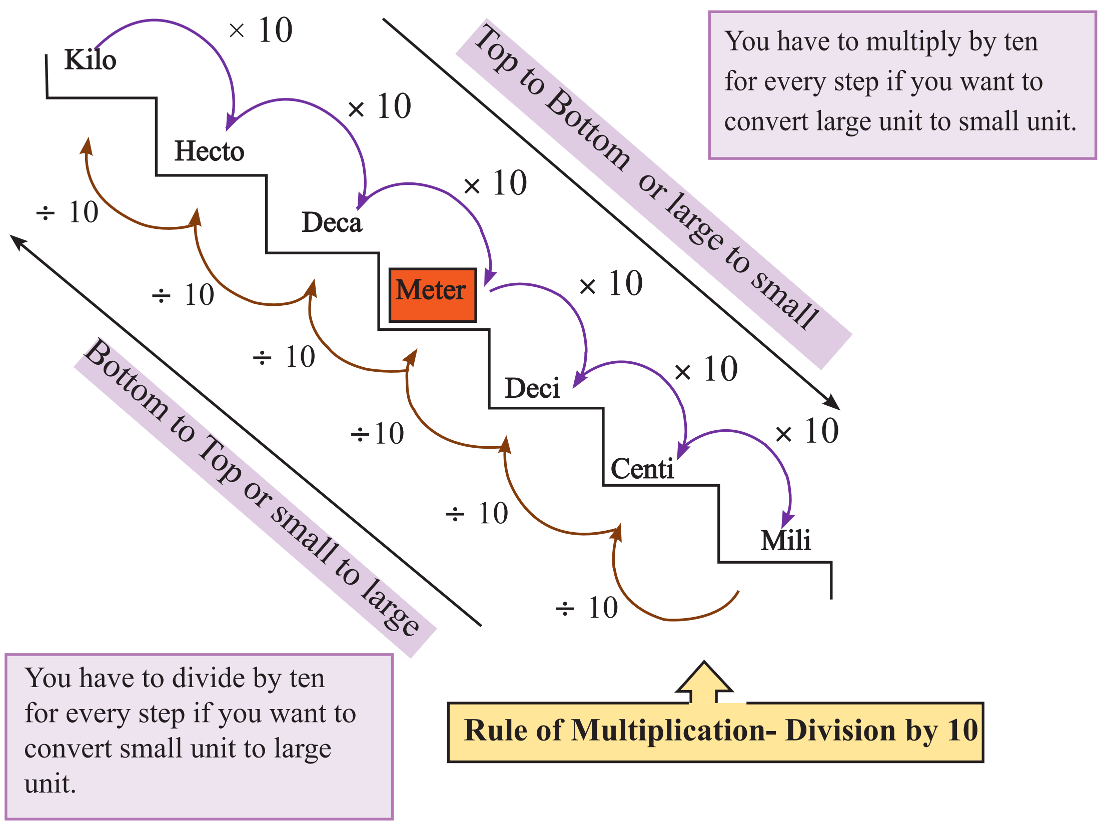
Then they went to another store for buying necessary commodities. They purchased 1 kg pulses, 1litre oil along with a few other things and returned home. Both Joya and Anik had one question in mind that is there any fixed amount with which a comparison is made while measuring this distance, length of wire and other things? If yes then what that fixed amount is called of measurement? Both of them thought that they would try to know this from their teacher in school. The standard physical quantity (in the Science

and Technology chapter of your Science book, there is a detailed discussion about the measurement of different quantities) comparing to which the measurement of other quantities are done is called the unit of measurement. Every measurement requires a standard compared to which the measurement of physical quantities are done. This standard is called the unit of measurement.

Measurement has been in practice since ancient time for doing daily and trade activities. This measurement had many local and area based units for different quantities. For example, just a few years back in our country we used mon, sher, chotak, tola as the units for measuring mass. Again, yards, feet, inches and miles are still in use for measuring length and distance. These units still might have been used locally. Since there were different measurement systems used in different countries for doing the measurement, various problems in international trade and commerce and in the exchange of scientific information began to arise. This is why the same standard of measurement was badly needed all over the world. From this it was decided in 1980 to introduce similar units of different quantities all over the world. This system of units is called International System of Units or in short SI.

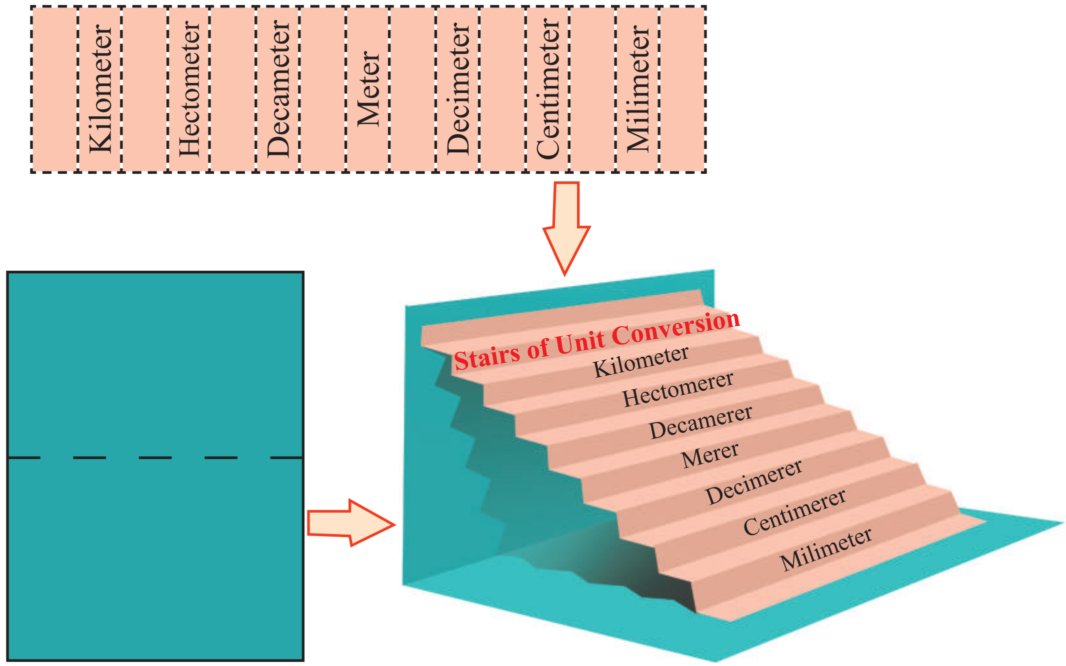
In this chapter, we will discuss about the unit for measuring length. There are two conventional systems for measuring length: 1) British System and 2) Metric System. In British system, yards, feet and inches are used as the units for measuring length. But nowadays, metric system is used in most of the countries of the world for measuring length. One of the features of measurement in this system is, it is tenfold/ ten times. In this system, measurement can easily be done through decimal fractions.

Conversion of Units to measure length in Metric System: Stair Method



Individual Task : Make stairs of Unit Conversion by paper.

(You can take help of the following diagram)



This method was first introduced in France in the eighteenth century. And it was introduced in Bangladesh on 1 July 1982. In this system, meter, centimeter and kilometer are used as the units of length measurement. You'll be surprised to know, scientists have defined the unit of length meter or centimeter differently at different times. The definition of metre got changed with the advancement of science. After a long 200 years of experiment, the scientist have defined 'meter' in 1983 as stated below: 'The length travelled by light in 1 share of 29,97,92,458 shares of a second.' Modern science has recognized this definition of meter as the most prime definition. You will find out more about this when you get to the higher classes.

Pair Work: Students will measure each other's height by using scale or measuring tape.

- At first, guess and write down the height of a peer in any of the units.
- Now, express the height, you got earlier by measuring with scale or measuring tape, in centimeters, meters and feet.
- Determine the difference, if found, between the guessed and measured heights.

Individual Work: Measure the length, width and height of the math textbook in inches and centimeters by using a scale. Make a table in your exercise book and write down the findings. Now, observe the table and verify the relationship between inches and centimeters.

Math Textbook	Original Length (measured with purchased scale)		
	Inches	Centimeters	Relationship between inches and centimeters
Length			
Width			
Height			

Group Work : Measure the length and width of your classroom and any of the stairs situated between two floors of the school with measuring tape. Now, fill up the table below with the findings.

Our classroom and stairs							
		Approximate Measurement	Actual length measured by purchased scale, tape or any other measuring instrument				
			Yard	Feet	Inch	C.M	Meter
Our classroom	Length						
	Width						
Stair of school	Length						
	Width						

Measuring the thickness of a 2 taka coin by centimeter scale

Materials: Few 2 taka coins, Centimeter scale

System 1:

a) Guess the thickness of a 2 taka coin.

b) Write down the thickness in exercise book.

c) Now, measure the thickness of the coin in centimeter scale as shown in the picture at right side.

d) Find out the difference, if any, between the guessed and measured value of thickness.

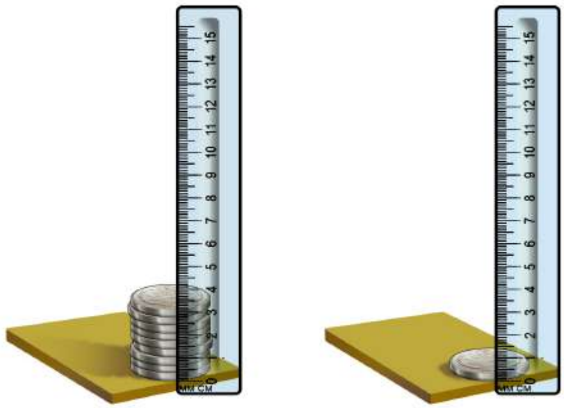
System 2 :

a) Assemble a few coins, putting one on the top of the other, as shown in the picture.

b) Now, measure the length of the assembled coins with centimeter scale.

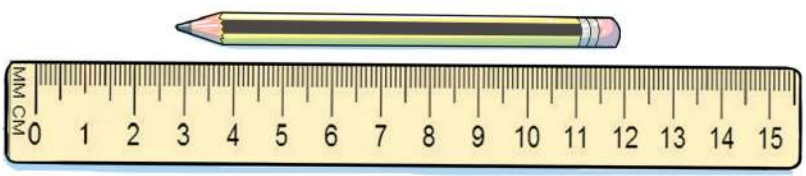
c) Divide the total length with the total number of coins and you'll get the thickness of one coin.

Measure the diameter of a circular coin in various ways.

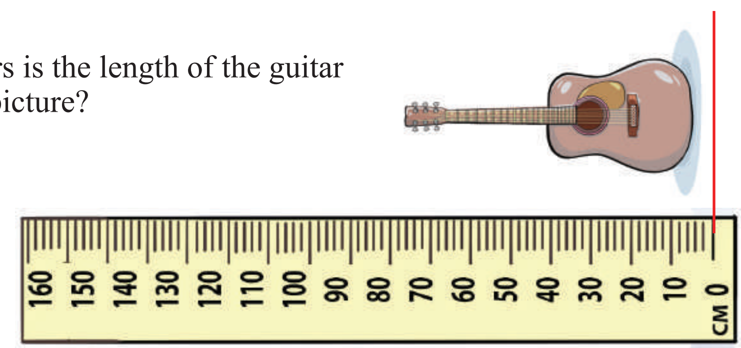


Exercise

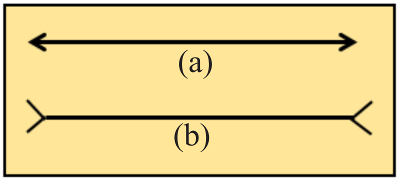
1. What is the length of the pencil as shown in the picture?



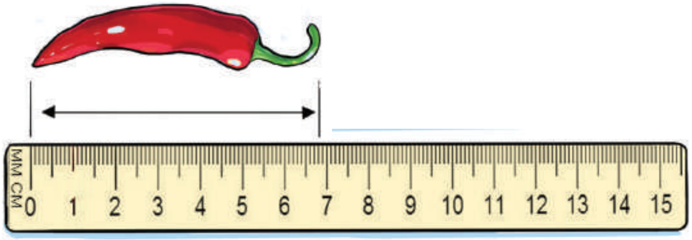
2. How many meters is the length of the guitar as shown in the picture?



3. Which of the following lines is longer? Guess it. Now, measure the lines A and B in centimeters and verify your guess.



4. Determine the length of the chili in centimeter and in millimeter. Then express the length determined in milimeter to centimeter.

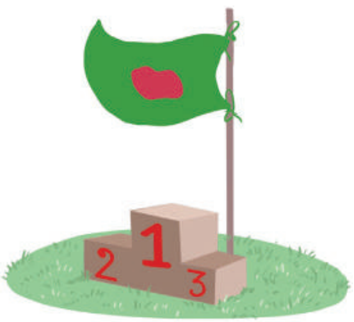


5. The distance covered by 5 students of class six in the long jump event of school's annual athletic competition is given below:

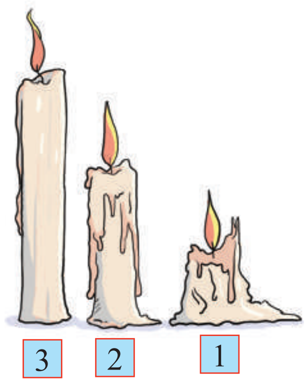
Student's Name	Distance Covered
Sadia Islam	3.50m
Shuborna Roy	4.05m
Monika Chakma	4.50m
Adiba	3.80m
Rina Gomez	3.08m

a) Express the covered distances in meter and in centimeter.

b) Which three students will stand in the 1^{st}, 2^{nd} and 3^{rd} positions in the victory stand to show respect to the national flag?



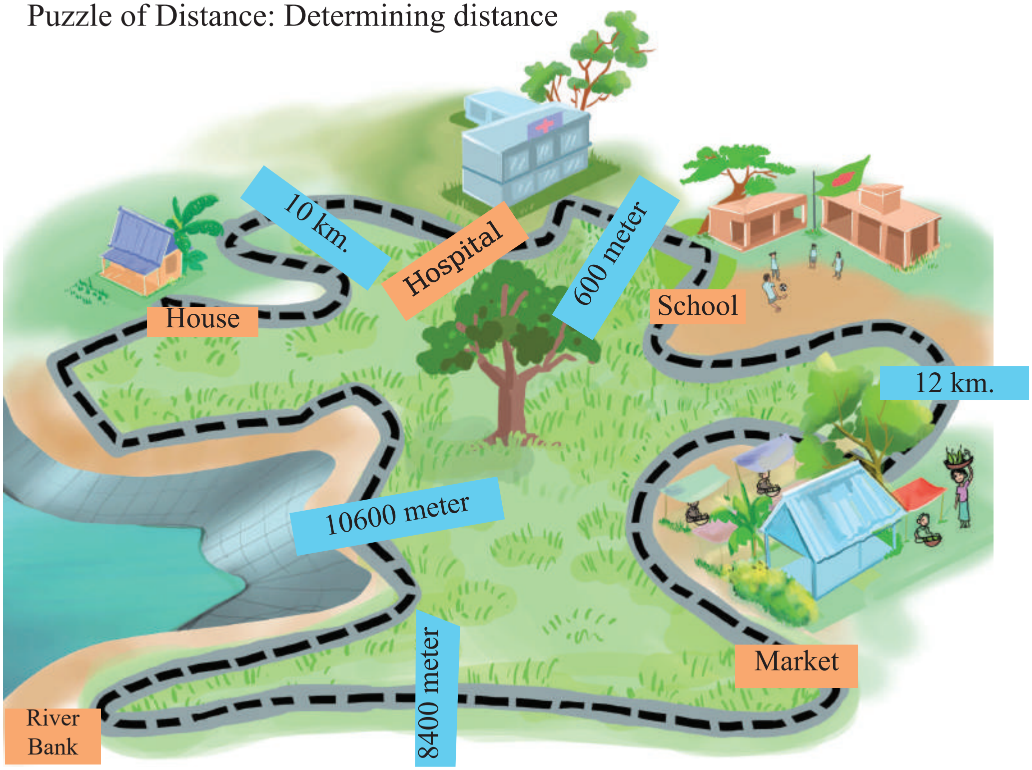
6. Draw pictures of three candles, having different heights, with flames as shown below. Measure the drawn candles and fill up the following table.



Whose length has to be measured	Approximate length	Length (cm and mm)	Length (cm)
Candle- 1			
Flame-1			
Candle- 2			
Flame-2			
Candle- 3			
Flame-3			

7. Measure the diameter of a marble in centimeters and inches by using centimeter or inch scale.

8. Puzzle of Distance: Determining distance



a. Which paths lead to the market from home? Determine the distance of each path and find out the path having the shortest distance.

b. Which paths can lead to the hospital from the riverbank? Determine the distance of every path and find out the shortest one.

Peer evaluation rubrics for measuring the length and width of the classroom and the distance between two stairs of the school

During the group work, complete the assessment after observing your group members' activities.

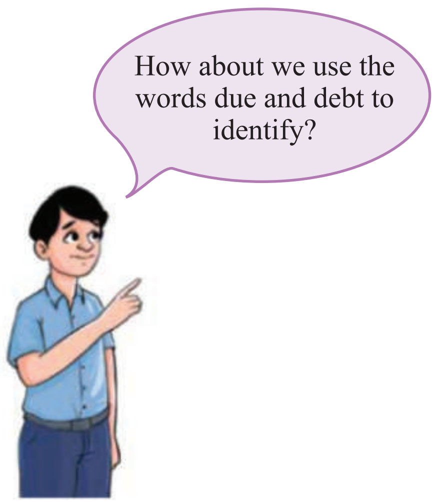
The activities (while assessing) you need to observe are listed at the left column in the following table. If you don't understand anything then get it clarified from the teacher. You need to fill up the table for every group member of your group. At first, write the names of the team members in the allotted box below. You'll understand the peer evaluation better from the following example. Suppose a member of your group, "Mita", completed a task mentioned in the lower left column - "Determine the width of the classroom in meters". Give her three stars ☆☆☆ if she has completed the task. And if she completes it partially then give her ☆☆ two stars. And if it happens that Mita has measured but the result is not correct then give her a single ☆ star. Again, if she doesn't participate in the group work, then you'll write, 'Not Participated'.

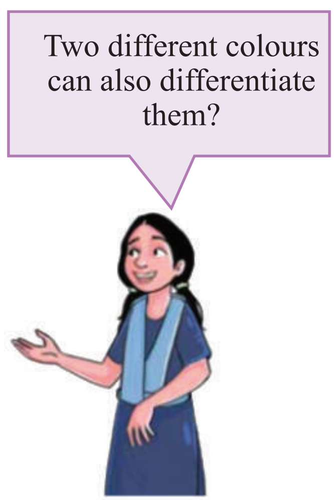
<table>
<tr><td>Finished Completely
☆☆☆</td><td>Finished Partially
☆☆</td><td colspan="3">Measured but the result is not accurate
☆</td><td colspan="3">Not Participated</td></tr>
<tr><td colspan="2">Assessment Field</td><td colspan="6">Names of Group Members</td></tr>
<tr><td rowspan="3">Measuring the length of the classroom</td><td>The method of measurement using tape has been followed accurately</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Determined the value in different units (yards / feet / inches / cm / m) while measuring length</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Determined the relationship between the values of different units</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Measuring the width of the classroom</td><td>Can follow the method measuring the width by tape.</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="4">Measured the length of the stair</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="4">Measured the width of the stair</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="4">Measured the distance between the two stairs</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">How did you feel doing this peer evaluation? What kind of problems did you face? Write those in the side column.</td><td colspan="6"></td></tr>
</table>

The World of Integers

Anita went to market for buying a pen. She only got 10 taka. But the pen costs 15 taka. The shopkeeper wrote 5 taka as the due amount to be paid by Anita. He wrote 5 taka next to Anita's name in his account book just to remember later on. Ratul also came to the shop at that time to buy the pen. To buy the pen he gave the shopkeeper a 20 taka note. As there was no change, the shopkeeper asked Ratul to take 5 taka some other time. Again, he wrote 5 taka next to Ratul's name in his account book just to remember later on.

But a problem occurred. How will the shopkeeper remember that Anita has to pay him 5 taka and he needs to pay Ratul 5 taka? Can you solve this problem of the shopkeeper?

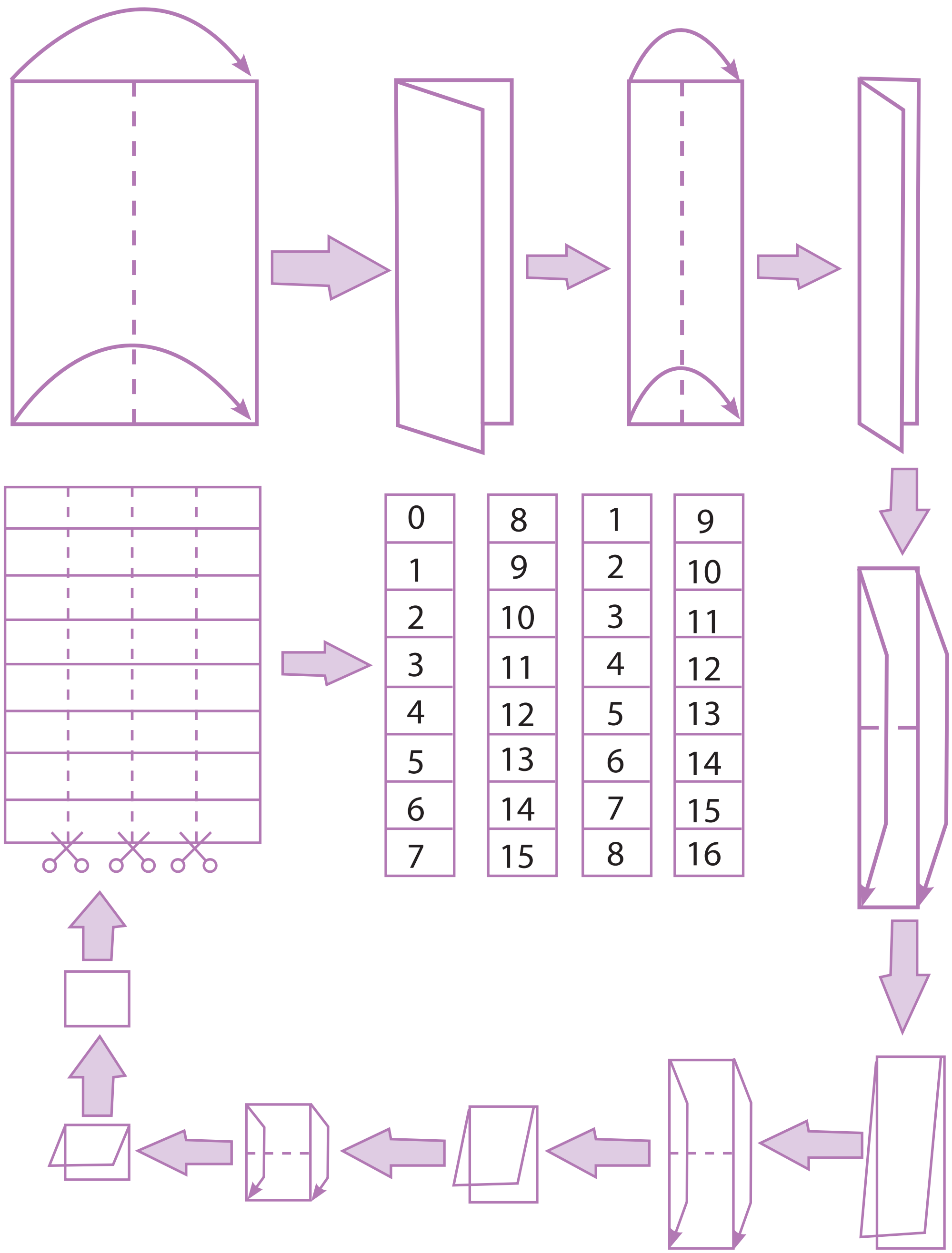




Afterwards, Anita and Ratul went to school and there they faced a similar problem while playing a game named, “One pawn, two players.” Let’s observe how they got the solution of that problem.

One pawn, Two players

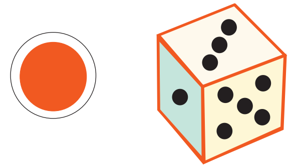
❖ At first, Anita and Ratul folded an A4 sized paper as shown in the following picture. Then they cut the paper into four strips and wrote down the numbers as shown below.



- Then they took two strips of paper and arranged those as shown below.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

- To play the game, one pawn and one dice will be required.



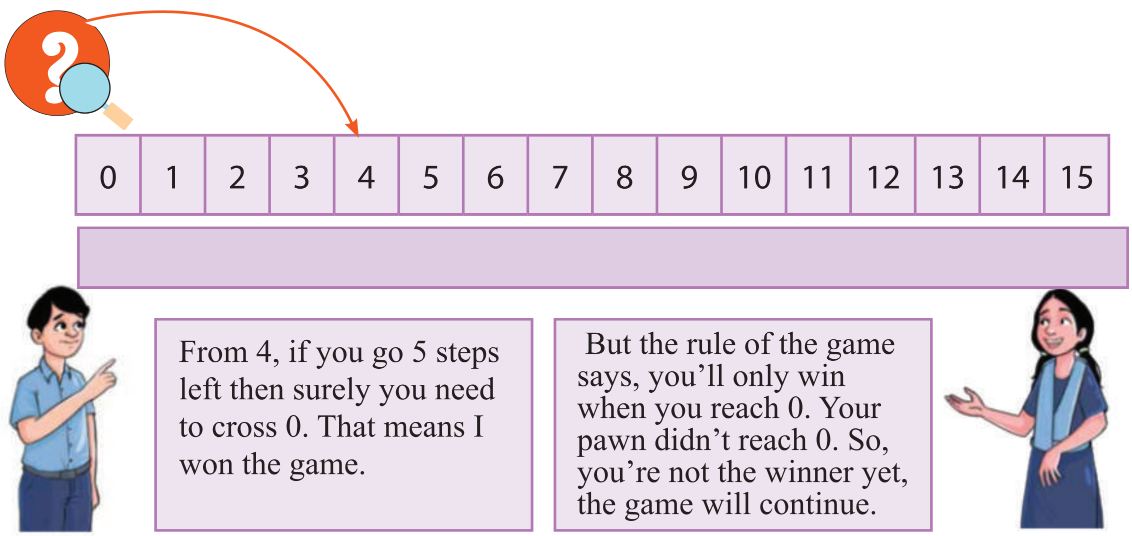
In the opening, they only kept a pawn on 8.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

After this, the rule of this game is quite similar to playing Ludu.
But there are two dissimilarities:

1) Here, there's only one pawn.
2) For the first roll of the dice, the pawn must go to the right side. And for the second roll of the dice, the pawn must go to the left side. For both, the pawn will move to right or left as per the drawn number of the dice. After that, the first one will roll the dice again and thus the game will continue.

The first one will be the winner once she reaches 15 and the second one will be the winner if he reaches 0.
So, Anita rolled the dice first and then Ratul. Then again Anita and thus the game continued.
At one point of the game the pawn was on 4. In such condition, Ratul rolled the dice and got 5. Now, in which direction Ratul will place the pawn? There is nothing at the left of 0.



But how will the game continue? There's no number at the left of 0. Then, Anita and Ratul cracked an idea. They took the two paper strips and placed those at the left of 0. Now, when Ratul got 5 from rolling the dice, he could go 1 step left to 0 and could place his pawn.

Ratul got 5 from rolling the dice

8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8

8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8

But now it's seen that, at the right and left side of 0 there is repetition of the same number. So to differentiate, they painted the numbers appearing at the left of 0 with green colour. Then they changed the rules of the game a little and started playing again.

Now, at the beginning the pawn will be on 0.

The rules for winning will be the same for the first person, that is, s/he will be the winner only if s/he reaches 8.

But there will be new rules for the second person.

If the second person can reach 8 to the left of 0, that is, the green 8, then s/he will be the winner.

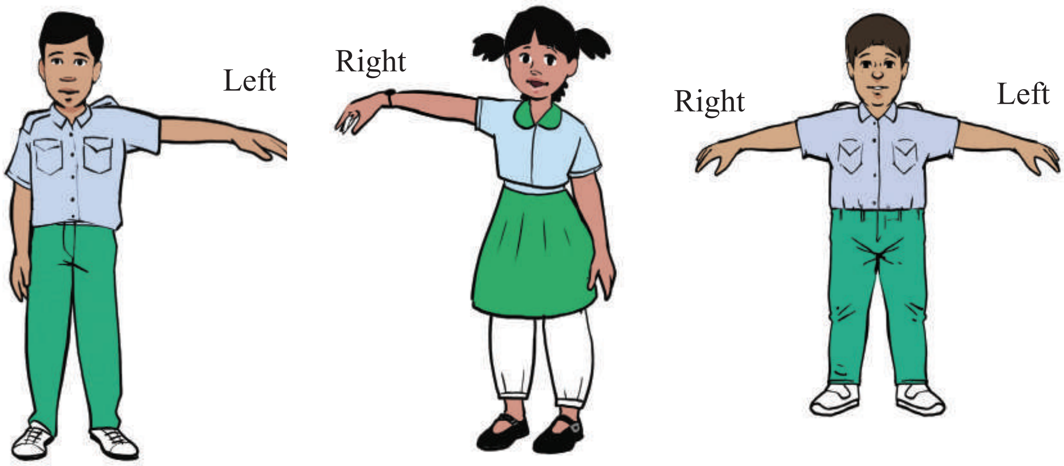
8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8

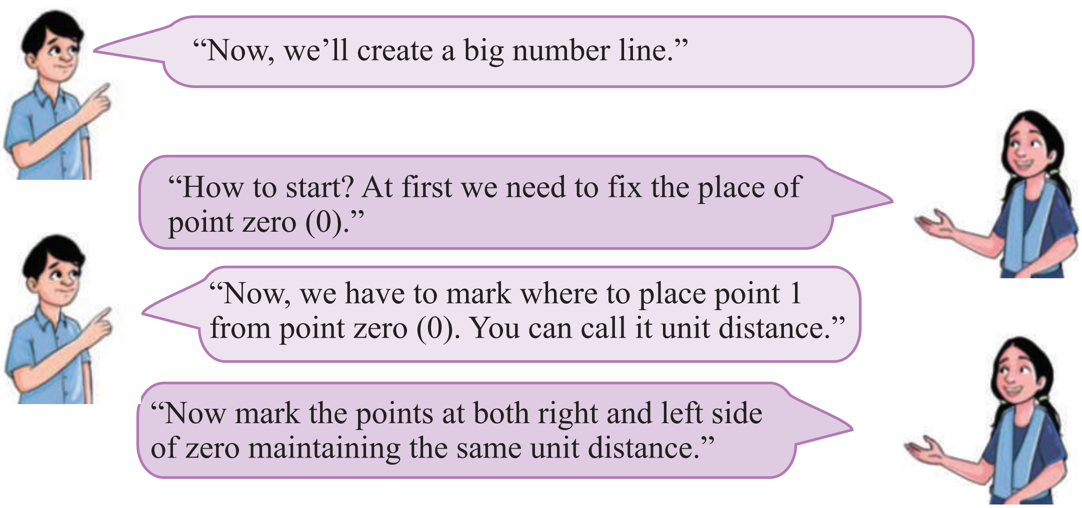
Then one day while playing they could not find green colour and started to wonder if the numbers to the left of 0 could be distinguished in a simpler way. Eventually they agreed that the numbers would be preceded by a minus sign or a negative sign '-'. As these numbers are appearing at the left of 0 so they will be smaller than zero. And we call these numbers as Negative Numbers.

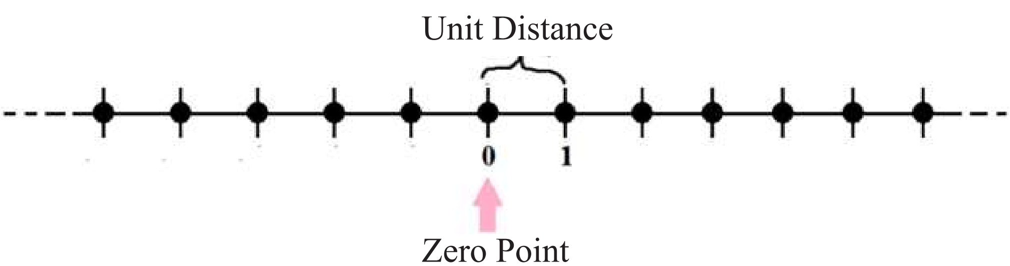
− 8	− 7	− 6	− 5	− 4	− 3	− 2	− 1	0	1	2	3	4	5	6	7	8

Integer Game on the Number Line

"Let's play a fun game today. At the beginning, all must stand up and extend their right hand to the right side. Now, lower down the right hand and extend the left hand to the left side. Please remember to keep your head and legs in a static position. Now, extend both the hands (right hand to the right and left hand to the left). If the right hand, relative to the head, is named as positive then what the left hand, relative to the head, would be called?"



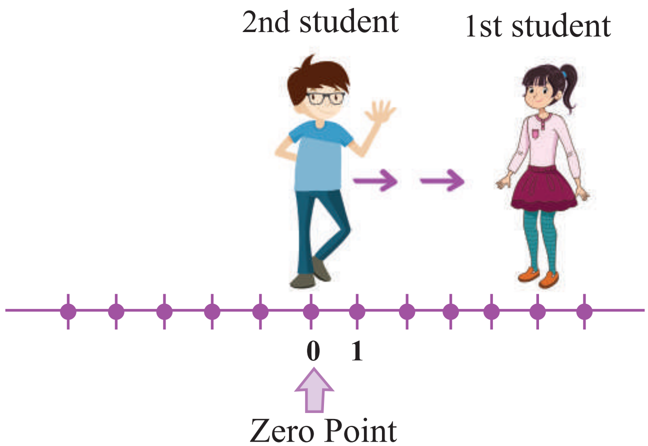




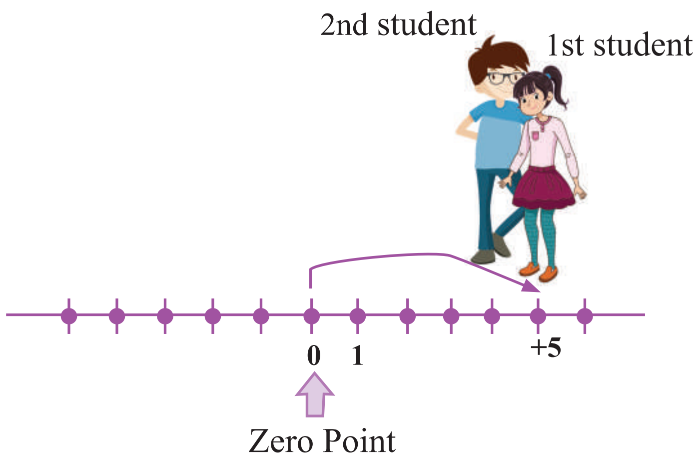
Now the game will begin. The game will be played in pairs. One of the students from each pair will stand on one of the steps of the number line according to his choice. The other student in the pair will stand on the point of zero (0) and will advance one step at a time, to where the first student of the pair is.

And then he will write the position of the first student with figure. Here, mark the steps appearing at the right of zero (0) with '+' and the steps appearing at the left of zero (0) with '-.'

'The first student of the pair has crossed 5 steps at the right; therefore mark his position with +5.'



'The student has crossed 4 steps at the left; so mark with -4.'



Fill up all the steps of your number line with numbers and signs through playing the game.

- Write down each step below with a '+' or '-' sign depending on their position:
  - a) 4 steps to the left of zero
  - b) 7 steps to the right of zero
  - c) 11 steps to the right of zero
  - d) 6 steps to the left of zero

Decrease and Increase of Numbers

The Game on the number line tells you that the numbers to the right of zero are positive but the numbers to the left are negative. If you go 1 step to the right of a number, you will get the next number of that number and if you go 1 step to the left, you will get the previous number of that number.

1) Fill in the table below by writing the numbers before and after the given numbers:

Previous Number	Given Numbers	Next Numbers
	10	
	8	
	– 5	
	3	
	0	
	– 1	
	– 2	
	1	
	– 10	

Use of negative numbers

The use of positive and negative numbers in real life is discussed below:

Income, Expense

Profit, Loss

Increase, Decrease

These are our familiar words. The first of the pair is the opposite of the second.

Income, profit and increase mean growth in quantity.

Again, expense, loss and decrease reduce the quantity.

If 5 taka income is marked with +5 taka then 7 taka expenditure can also be marked with -7 taka.

Similarly, +6 means profit of 6 taka whereas -8 means loss of 8 taka.

Observe from the above discussion that, to clarify the difference between two similar quantities having opposite directions, if we mark one with (+) sign then the other will be marked with (-) sign.

(+) marked quantities are called positive quantities and (-) marked quantities are called negative quantities. For this reason (+) and (-) symbols are called positive and negative symbols.

Game of Antonyms

There are some words and their opposite words given in the table below. Fill in the table with few more words you know and with their opposites. Now express the words in each row of the table and the opposite pairs of words as per your wish through positive (+) and negative signs (-).

(Here, you may consider any word of the pair as positive. But in that case the opposite word of that word must be negative.)

Word		Opposite Word	
Big	+	Small	–
Light	–	Heavy	+
Income		Expense	
Left		Right	

1) Write a phrase that has the opposite meaning for each of the following phrases:

Given Phrases	Phrases with opposite meaning
Increase of weight	Decrease of weight
30 km north	
Market is 8 km north from home	
700 taka loss	
100 meters above sea level	

2) Write the numbers mentioned in the following sentences with appropriate signs:

a) An Airplane is flying two thousand meters above the ground.

b) A submarine is moving at a depth of eight hundred meters from the sea level.

c) Depositing two hundred taka in bank.

d) Taking a loan of seven hundred taka from the bank.

Integer

The numbers 1,2,3, ... were first discovered for the needs of the humans.

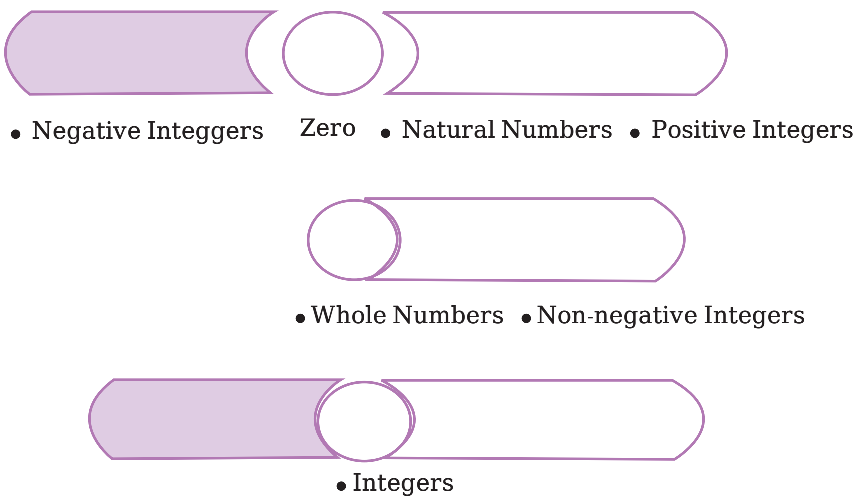
These are called natural numbers or positive integers. If we take 0 with normal numbers then we get 0,1,2,3.........which are called whole numbers or non-negative integers.

Again, $-4, -3, -2, -1$ these numbers are called negative integers.

Combining negative integers and non-negative integers, we get,

$-4, -3, -2, -1, 0, 1, 2, 3,$........These numbers are integers.

Numbers can be reresent by the following diagram

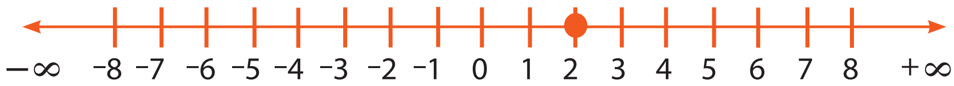


Placing integers on number line (Determining the place of integers)

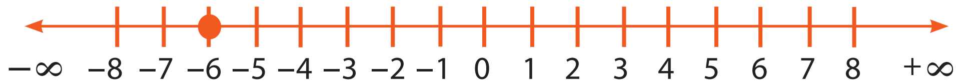
Draw a straight line and take a point 0 on it.

So, the point 0 divides the straight line into two parts. One part extends limitlessly to the right and the other limitlessly to the left. The right side of 0 is considered positive and the left side is considered negative. Take a certain length as the unit. Mark points at both right and left side of 0 maintaining equal distance. Mark the points appearing at the right side of 0 alternately with $+1, +2, +3, +4.$ ..or only 1,2,3,4... and the points appearing at the left side with $-1, -2, -3, -4.$ Use $+\infty$ sign at the right and $-\infty$ sign at the left side to denote infinity or limitlessness.

Now, to place a positive integer 2 on the number line, enclose the point with a deep circle that is 2 units to the right of the 0 point .The enclosed point with the deep circle will be the place of 2.



Again, to place a negative integer −6 on the number line, enclose the point with a deep circle that is 6 units to the left of the 0 point. The enclosed point with the deep circle will be the place of −6.

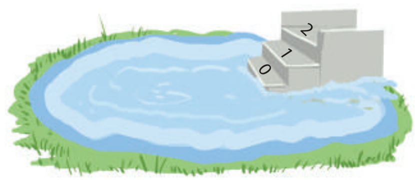


- Now, place the following numbers on number line:

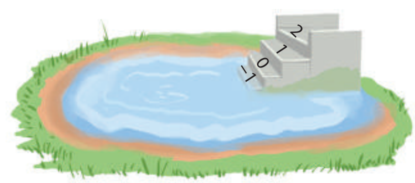
  a) +5   b) −10   c) +8   d) −1   e) −6

Order of Integers

There is a stairway pond in the village where Rama and Rani live. There are 10 stair steps from the side of the pond to the water level. One day, after reaching the pond-side they discovered that the water is only 5 stair steps away. They marked the current level of water with 0 to see up to where the water level rises during the rainy season. They also marked the top steps with 1,2,3,4,5. After the rains in the monsoon they saw that the water level rose up to 3 steps. A few months after the rainy season they saw that the water level dropped 3 steps below the 0 mark. They pondered over the ways to mark the steps below. Can you think of any suggestion that you can share with them ?



At last they thought that since the water level goes down once water decreases, then and there they thought that the integers smaller than 0 is called negative integers. So, as they've marked the current level of water with 0, anything coming after that 0 should have a (-) minus sign and it would be much helpful. Accordingly, they marked the steps below 0 with −1, −2, −3. The water level decreased more after a few days. Then, they marked that step with −4.



So, it is seen that, $-4 < -3$. Similarly, it can be said that, $-5 < -4$.

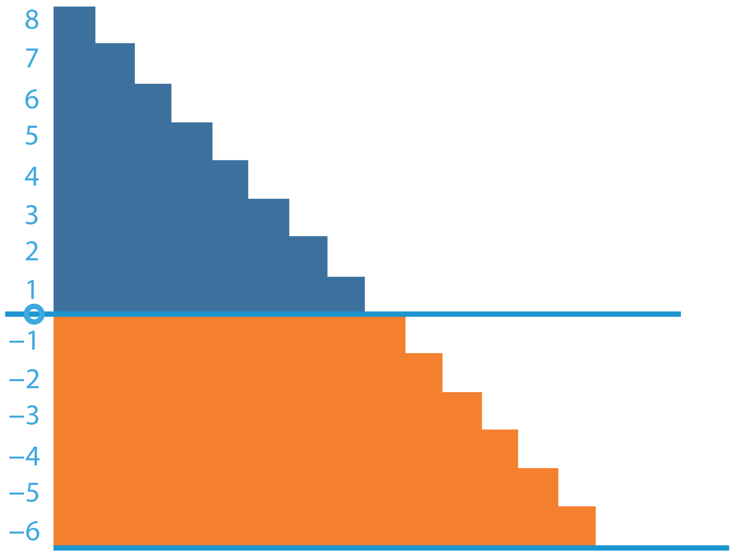
Let us once again place the integers on the number line:

We know, $7 > 4$ and in the number line we see that 7 is in the right of 4.

Similarly, $4 > 0$ that means 4 is in the right of 0. Again, 0 is in the right of -3, so $0 > -3$. Similarly, -3 is in the right of -8 so $-3 > -8$. Thus we see that the value of the number increases when we go to the right of a number line and the value decreases when we go to the left of a number line. Therefore $\ldots -3 < -2,\ -2 < -1,\ -1 < 0,\ 0 < 1,\ 1 < 2,\ 2 < 3, \ldots$

That is, we can write the integers alternately in the form of $\ldots -4, -3, -2, -1, 0, 1, 2, 3, \ldots$

1) Look at the following picture.



Fill in the blanks below by taking idea from the picture $<$ or $>$ by using signs.

(a) $-1 \square 1$ (b) $0 \square -1$ (c) $-4 \square -9$

(d) $-1987 \square -2999$ (e) $-64 \square -59$, (f) $9 \square -9$

(g) $-57 \square -59$, (h) $-2 \square -159$

2) Arrange the numbers $-5, 7, 8, -3, -1, 2, 1, 0, 9, 3$ in ascending order with the help of number lines.

3) The temperature of four different places, on any given day, from various countries are listed in the following table:

Name of place	Temperature	Empty Column
Dhaka	0^0C above 30^0C	
Kathmandu	0^0C below 2^0C	
Sreenagar	0^0C below 6^0C	
Riyadh	0^0C above 40^0C	

a) Write the temperature of different places in the blank column above using integers with appropriate signs.

b) The numbers on the number line below indicate temperature.

(i) Write the names of the above places on number line according to the temperature.

(ii) Which place is the coolest?

(iii) Write the names of places where the temperature is above 10^{0}c

Show on the number line which of the following numbers will appear to the right of the other :

2, 9

− 2, 5

0, − 1

− 11, 10

− 6, 6

5) Write the integers between the given numbers in ascending order:5)

( ) 0 and −7 ( ) − 4 and 4

( ) 0 and 7 ( ) 30 and − 23

) Write four negative integers greater than -20.

a) Write four negative integers smaller than -10.

b) Write four negative integers between -10 and -5.

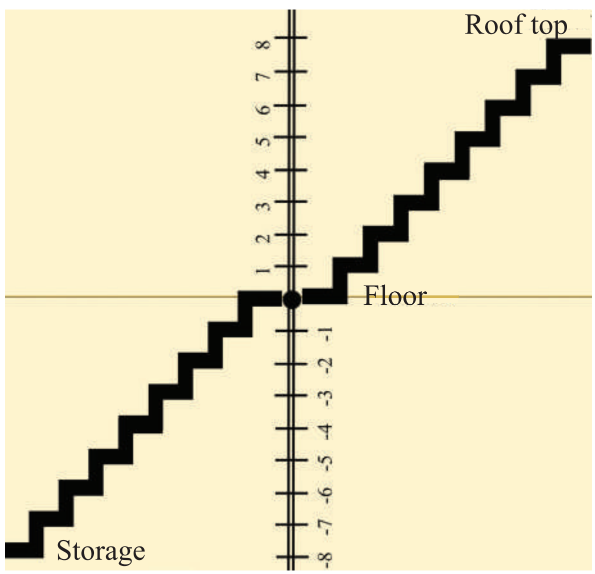
) Mark (√) for true and (×) for false next to the following sentences. If the given sentence is false, write it correctly.

Given Sentence	Is it true?	Correct Sentence (If the given sentence is false)
-4 appears in the right of -10 on number line	√	
-70 appears in the right of -10 on number line	×	
The smallest negative integer is -1		
$-\ 26$ is greater than $-\ 20$		
The number -25 is located between the numbers -5 and 15		
0 is a positive number		
0 is a negative number		
A negative number is greater than any non-negative number		

Addition of Integers

In Tarek's one storey building, there is a staircase to go to the roof and to the warehouse below. Each stair leading upward from the floor of the house is a positive integer, and all the stairs going down to the warehouse is a negative integer.

The ground floor indicates 0.



a) Now read the following sentences and fill in the blanks. (Two are done for you)

a) If ascended 6 stairs above the ground level then,

$\boxed{+6}$

b) If descended 5 stairs down from the ground level and then ascended 7 stairs up,

$\boxed{(-5) + 7 = 2}$

c) If descended 4 stairs down from the ground level,

$\boxed{}$

d) If ascended 2 stairs up from the ground level and then ascending 3 stairs more from there,

$\boxed{}$

e) If descended 4 stairs from the ground level and then descending 2 more stairs there,

$\boxed{}$

f) If descended 5 stairs down from the ground level and then ascending 3 stairs up,

$\boxed{}$

g) If ascended 4 stairs from the ground level and then descending 8 stairs from there,

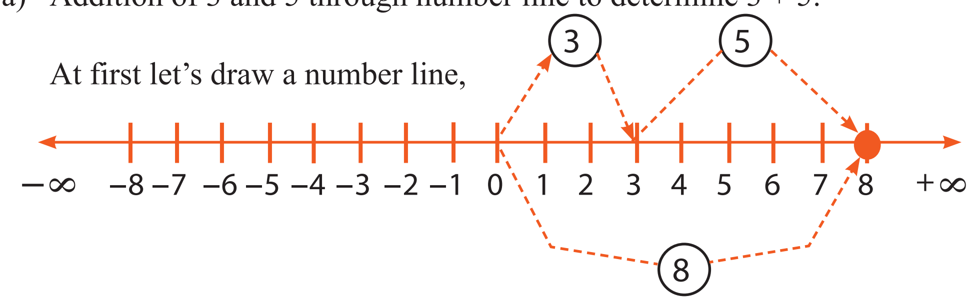
$\boxed{}$

- Draw a number line in group and prepare few questions and answers like seen above. Follow teacher's instruction while exchanging and evaluating the questions and answers with other groups.

Addition of integers with the help of number line

a) Addition of 3 and 5 through number line to determine 3 + 5:

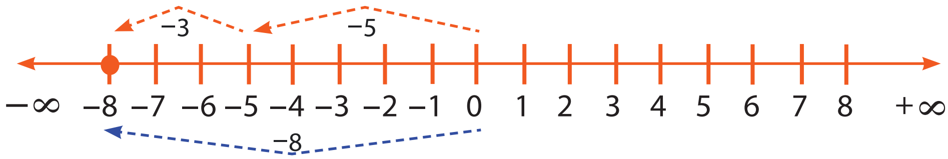
At first let's draw a number line,



Let's go 3 steps right from the 0 point of number line to reach point 3. Then let's move 5 steps more right to reach point 8.

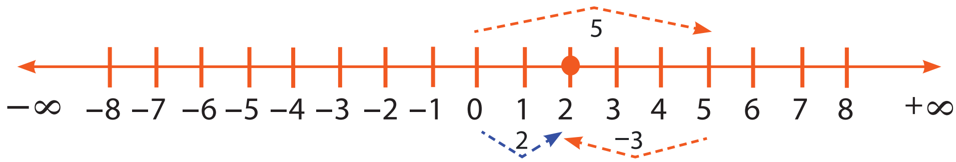
Then, the sum of 3 and 5 will be, 3+5=8

(b) Addition of -5 and -3 through number line that means, determining (-5) + (-3):
At first let's draw a number line,



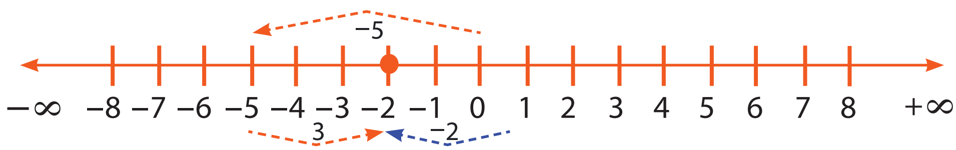
Let's go 5 steps left from the 0 point of number line to reach point -5. Then let's move 3 steps more left to reach point -8. Then, the sum of -5 and -3 will be, $(-5) + (-3) = (-8)$

(c) Addition of 5 and -3 through number line that means determining $5 + (-3)$:
At first let's draw a number line,



Let's go 5 steps left from the 0 point of number line to reach point -5. Then let's move 3 steps more left to reach point -8. Then, the sum of -5 and -3 will be, $(-5) + (-3) = (-8)$

(d) Addition of -5 and 3 through number line that means determining (-5)+ 3:
Let's draw a number line first,

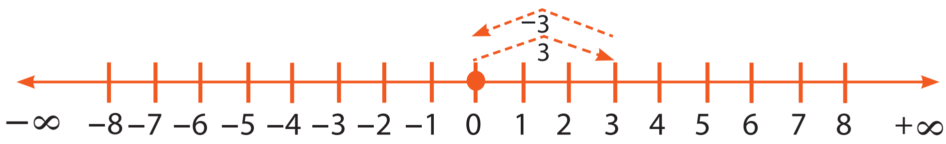


Let's go 5 steps left from the 0 point of number line to reach point -5. Then, let's move 3 steps right from there to reach point -2.
So, the sum of -5 and 3 will be, (-5) + (3)= -2

From the above discussion we observe that:

- If a positive integer is added to an integer, then the sum is greater than the integer.
- If a negative integer is added to an integer, then the sum is smaller than the integer.

Let's determine the sum of two integers 3 and −3. At first, let's move 3 steps right from the 0 point of number line to reach the point +3. Then, let's move 3 steps left from there. Which point did we reach?



From the above diagram we observe that, $3+(-3)=0$ that means we reached the point 0.

Therefore, we get zero by adding two integers 3 and (−3). That means, if a positive integer is added with its negative integer then the sum is zero.

In this case -3 is said to be the positive opposite of +3 and +3 is said to be the positive opposite of -3.

1) Write some positive and negative integers along with their positive opposites and then show them on number line.

2) Determine the sum of the followings using number line:

(a) $(-2)+6$ (b) $(-6)+2$ (c) $(-9)+6$

(d) $5+(-11)$ (e) $(-1)+(-7)$ (f) $(-7)+20$

3) Prepare two more questions like this on your own and solve those using number line.

Finding out the sum of multiple integers

Till now, you have observed the way of determining the sum of two integers. Now, let's find out the sum of multiple integers using this concept.

- In the beginning, we will determine the sum of −9,+4 and −6 these three integers. That means, we will determine, $(-9)+(+4)+(-6)$.

Solution: Arranging the negative integers side by side from the given integers we get,

$(-9)+(+4)+(-6)$

$=(-9)+(-6)+(+4)$

$=(-15)+(+4)$

$=-15+4$

$=-11$

- Now, we will determine the sum of +30, −23, −63 and +55 that means we will find out, $(+30) + (-23) + (-63) + (+55)$

Solution: Arranging the positive and negative integers side by side from the given integers we get,

$$(+30) + (-23) + (-63) + (+55)$$
$$= (+30) + (+55) + (-23) + (-63)$$
$$= (-63) + (-23) + (+30) + (+55)$$
$$= (+85) + (-86)$$
$$= 85 - 86$$
$$= -1$$

Now solve the following problems.

1) Determine the sum without using number line.

(a) 137 and −35 (b) −52 and 52

(c) −31, 39 and 19 (d) −50, −200 and 300

2) Determine the sum without using number line.

(a) $(+7) + (-11)$ (b) $(-13) + (-10)$ (c) $(+10) + (-5)$

(d) $11 + (-7)$ (e) $(-13) + (+18)$ (f) $(-10) + (19)$

(g) $(-1) + (-2) + (-3)$ (h) $(-2) + 8 + (-4)$ (i) $(-7) + (-9) + 4 + 16$

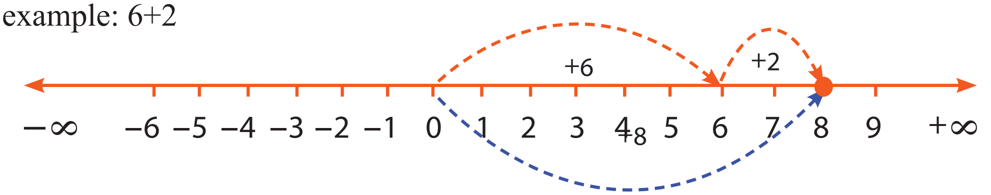
(j) $37 + (-2) + (65) + (-8)$ (k) $(-10) + 92 + 84 + (-15)$

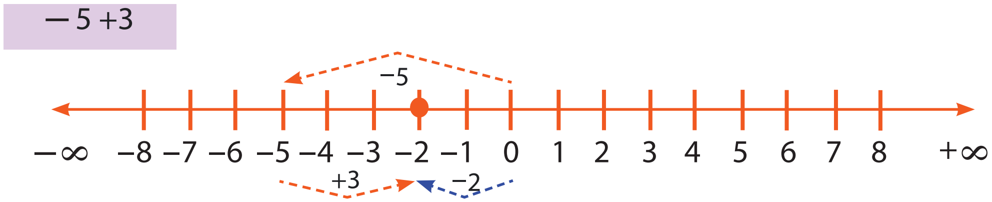
3) Prepare five more questions like this and solve those on your own without using number line.

Subtraction of integers with the help of number line

We have learned to add a positive integer to any number with the help of number line. Here we observed that, to add a positive integer to any number, we have to move towards the right side from the positional point of that very number.

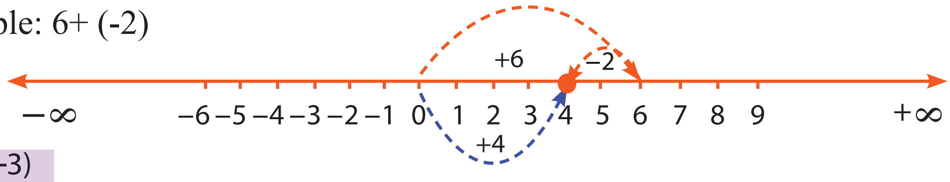
For example: 6+2



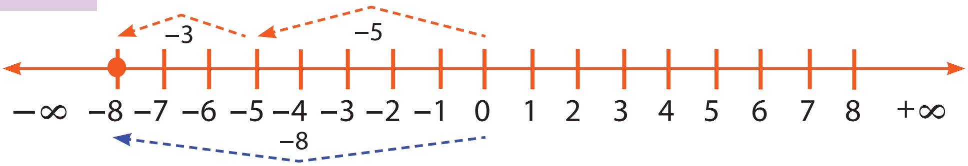


Again, to add a negative integer to any number, we have to move towards the left side from the positional point of that number.

For example: 6+ (-2)



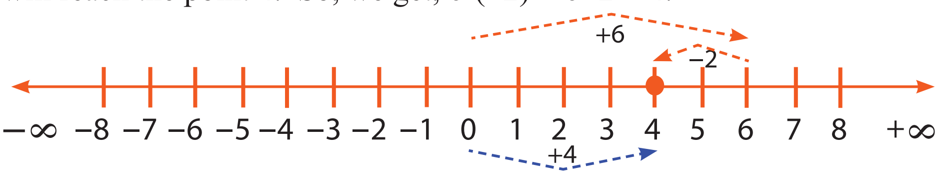
−5 + (−3)



Now, we will learn how to subtract an integer from another integer.

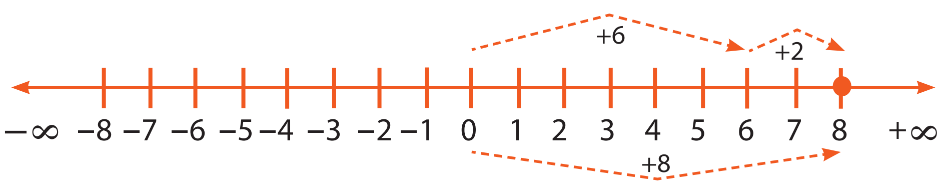
(a) Subtracting 2 from 6 with the help of number line, that means determining, 6-(+2).

For subtracting 2 from 6 using number line, we have to move 2 steps left from 6 and we will reach the point 4. So, we get, 6-(+2) = 6 -2 = 4.



(b) Subtracting -2 from 6 with the help of number line, that means determining, 6-(-2).

For determining 6−(−2), which direction do we go from the point 6, right or left? If we move two steps left from point 6, we will reach to point 4. Then we have to express-
6−(−2) = 4 ; but it is not correct.
We know, 6−2 = 4
So 6−(−2) ≠ 6−2
If moving 2 steps left from point 0 is '−2' then we can say moving '−2' steps left from 0 is actually moving 2 steps right from point 0.
So, 6−(−2) = 6+2 = 8
So we have to move 2 steps right from point 6.



Let's observe, −(−2) = +2 = 2

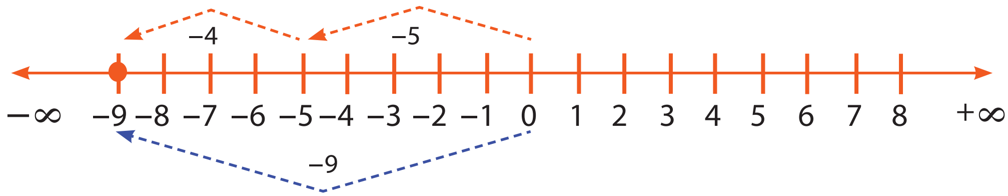
Let's consider the solution of the problem differently, we know that, the positive opposite of (−2) is 2. For this, the sum we get from adding the positive opposite of (−2) with 6 is equal to the result we get from subtracting (−2) from 6.

To subtract a number from the other means, adding the positive opposite of the second number with the first number.

So, we can write, 6-(-2) = 6+2 = 8.

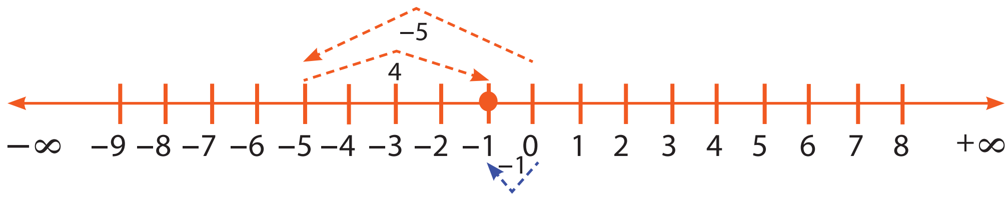
It is clear from the above example that, when a negative integer is subtracted from an integer, then we get a bigger number than the integer.

(c) Determining the value of -5-(+4) using number line.



So, we get, -5+(-4) =-9. So, -5-(+4) =-9.

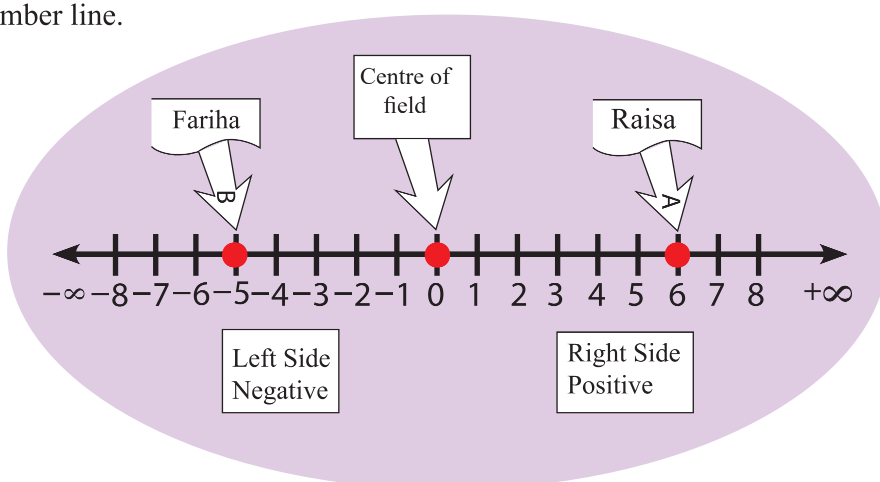
( ) Determining the value of -5-(-4) using number line.



So, we get, -5+4 =-1 so, -5-(-4) =-1.

Activity

1. Determine the value of -8-(10).
2. Subtract -4 from -10.
3. Subtract +3 from -3.
4. Two students of class six, Raisa and Fariha reach the points A and B respectively after crossing 6 steps right and 5 steps left from the center (point 0) of their school field. Right side represents positive.
   a) Write the positional numbers of A and B with markings.
   b) Show Raisa and Fariha's locations on number line.
   c) If Raisa and Fariha advance one step further then add their positional indicators using number line.



Exercise

1) What is the positive opposite of –a?

(a) + a (b) — a (c) $\frac{1}{a}$ (d) $\frac{-1}{a}$

2) If we add the positive opposite of 12 with 12 itself then the sum becomes-

(a) —24 (b) —12 (c) 0 (d) 24

3) if $\square$ — 15 = — 10 then what is the number in $\square$?

(a) — 25 (b) — 5 (c) 25 (d) 5

Answer the questions 4 and 5 considering the information given below:

−7, −8 and −9 are three integers.

4) If you add the positive opposite of second integer with the first integer then the sum is,

(a) — 15 (b) — 1 (c) 1 (d) 15

5) If the second integer is added with the sum of 1st and 3rd integers' positive opposites then the result becomes A,

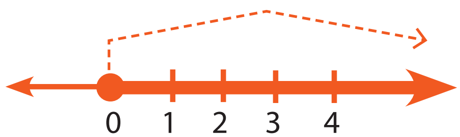
(a) A< —15 (b) A > —90 (c) A > 97 (d) A < —97

6) if A = 45 — (—11) and B = 57 + (—4) then,

(i) A = 56 (ii) B = —53 (iii) A — B = 3

Which of the following is correct?

(a) (i) & (ii) (b) (i) & (iii) (c) (ii) & (iii) (d) (i), (ii) & (iii)



7) The marked portion in the picture has

(i) Non-negative integer (ii) All prime numbers (iii) All even numbers

Which of the following is correct?

(a) i & ii (b) i & iii (c) ii & iii (d) i, ii & iii

8) Determine the subtraction.

(a) 35 — 20 (b) 72 — 90 (c) (—20) — 13

(d) (—15) — (—18) (e) (—32) — (—40) (f) 23 — (—12)

9) Use > or < or = in the blanks below.

(a) (−3) $(-3) + (-6) \square (-3) - (-6)$

(b) $(-21) - (-10) \square (-31) + (-11)$

(c) $45 - (-11) \square 57 + (-4)$

(d) $(-25) - (-42) \square (-42) - (-25)$

10) Fill up the blanks below.

(a) $(-8) + \square = 0$

(b) $13 + \square = 10$

(c) $12 + (-12) = \square$

(d) $(-4) + \square = -12$

(e) $\square - 15 = -10$

11) Determine the value.

(a) $(-7) - 8 - (-25)$

(b) $(-13) + 32 - 8 - 1$

(c) $(-7) + (-8) + (-90)$

(d) $50 - (-40) - (-2)$

12) $A = (-9) + 4 + (-6), B = 7 + (-4)$

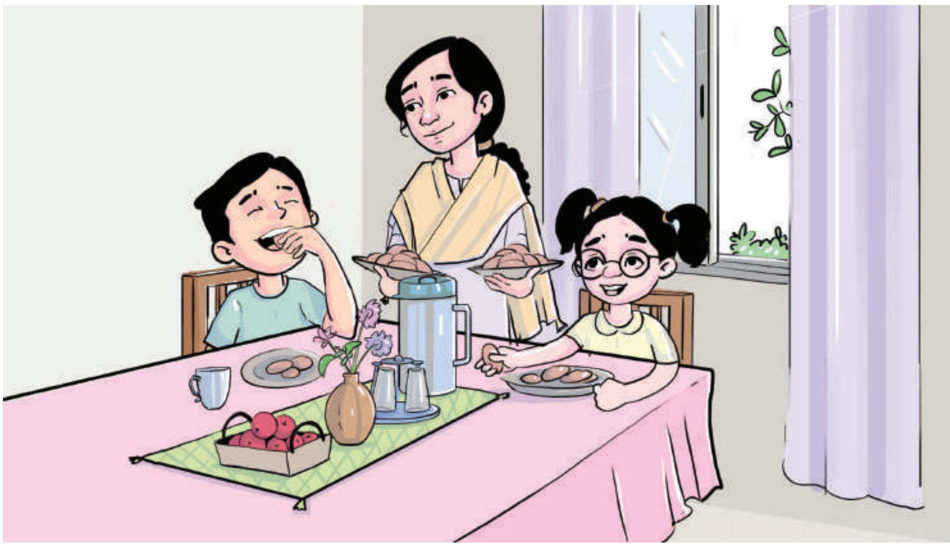
(a) Determine the value of B.

(b) Prove that $A < B$.

(c) Determine (A+B) after placing the value of A and B on number line.

The Game of Fractions

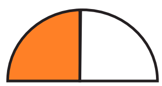
Ratul is a sixth grader student of Adarshagram High School. Ratul came to know about fractions in class four and five, therefore whenever it is required and possible, he calculates using his idea of fractions. We can divide things between ourselves easily using fractions. Again, fractions help us understand things that cannot be expressed by integers. For example, the other day Ratul's mother was making traditional cakes (Pithas) and there were five traditional cakes (Pithas).Ratul divided those five traditional cakes (Pithas) with his sister Riya. Riya is a student of class three. At first, Ratul gave Riya two traditional cakes (Pithas) and took the same amount for himself. And for the 5th one, Ratul divided it equally into two portions. Then, he gave Riya half of the portion and kept the other half portion for himself. Seeing this Pitha sharing process of Ratul and Riya, their mother became happy.



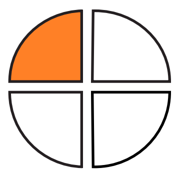
In which cases have you used the concept of fractions as done by Ratul above? Think and answer.

If we write the amount of Pithas, that Ratul and Riya got after sharing,in numbers then how it would be like?

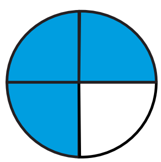
Ratul knew that we could write half of a Pithaas $\frac{1}{2}$. Then again while eating Pithas, Ratul asked Riya, if the half portion of the Pitha is again divided into two portions (as shown in picture 1), then what a portion would be of the whole Pitha?



Pic-1

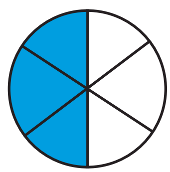


Pic-2



$\frac{3}{4}$

Pic-3



$\frac{3}{6}$

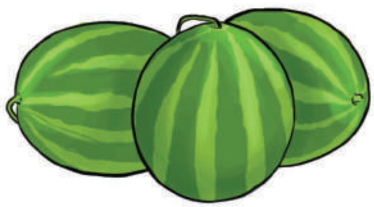
Pic-4

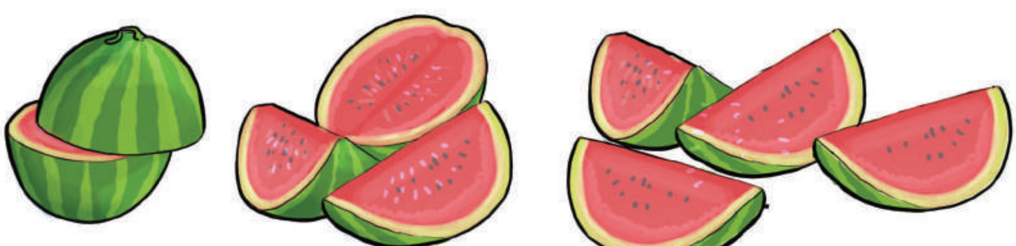
Hearing this question from Ratul, Riya divided her half portion Pitha into two portions and kept those besides Ratul's Pitha. It is observed from there that if four equal parts are kept together then it represents one whole Pitha (as shown in picture 2). Therefore, we can say that, each of the portion here is one fourth of the full sized Pitha that means $\frac{1}{4}$. Again, if all these four portions are kept together then we get $\frac{4}{4}$ or 1wholePitha. Riya and Ratul continued their discussion while having Pithas.If we take three equalportionsfrom four equal portions of a Pitha then we will say $\frac{3}{4}$ (as shown in picture 3). And if we take three equal portions from six equal portions of a Pitha then we will say $\frac{3}{6}$ (as shown in picture 4).

Riya then thought that fraction is a type of number that helps us in expressing parts of a whole thing. Ratul also thought that, to express a fraction the whole thing needs to be divided equally just like they divided the *Pitha* into two parts at first and then made four equal parts out of that.

Fraction is a number that represents parts of a whole thing. For example: In the above picture, (Picture-3) $\frac{3}{4}$ is a fraction that we call as "Three out of four." Here, 4 means equal parts of the whole Pitha or number of portions and 3 represents the extracted portions. In mathematical language, 4 is called as Denominator and 3 is called as Numerator.

Let's suppose, you along with 5 other friends have purchased three watermelons of same size. Then, those watermelons are being cut into pieces as shown in the pictures below.





Now, write your name and your 5 friends' names. Now, consider one watermelon as whole or 1part. Then, fill up the empty boxes writing the amount, in fractional form, each of your friends got as shown through the pictures.

Your name

____

Watermelon you got

$= \frac{\square}{\square}$

Name of friend 3

____

Watermelon he got

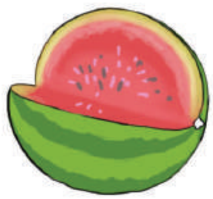
$= \frac{\square}{\square}$

Name of friend 1

____

Watermelon he got

$= \frac{\square}{\square}$



Name of friend 4

____

Watermelon he got

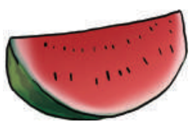
$= \frac{\square}{\square}$

Name of friend 2

____

Watermelon he got

$= \frac{\square}{\square}$



Name of friend 5

____

Watermelon he got

$= \frac{\square}{\square}$

Picture-5

Now, if you are asked who among your 5 friends was given more watermelon?

You can easily find the answer to this question if you can play the following game maintaining all the rules.

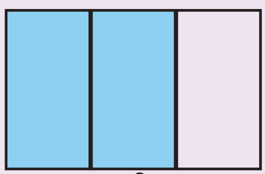
Name of the Game : Comparison of Fraction

Necessary Materials: checkered paper, colour pencil

Instruction: You can take help from your teacher in the classroom to understand the steps of the game. If you want to play the game in home, you can take help from your father/mother/ elder siblings to understand the steps.

Steps of the Game:

- Cut two strips from the checkered paper. Then divide a strip into three equal parts and paint two parts. That means, you'll paint $\frac{2}{3}$ parts only. Similarly, you'll divide another strip into four equal parts and will attach three parts in your exercise book and will paint those. That means, you'll paint $\frac{3}{4}$ parts only.(Follow the image below)
- Now compare the two painted parts- which one is bigger and which one is smaller. You'll fail in doing the comparison as the painted parts and the divided parts are different for both the strips.
- Now, draw two equal sized rectangular tables. Name the tables as Table A and Table B. If necessary, follow the teacher's instruction. Then, divide the Table A vertically into three parts and paint two parts (that means, $\frac{2}{3}$ parts. Again, divide Table B into four parts horizontally and paint three parts (that means, $\frac{3}{4}$ parts).



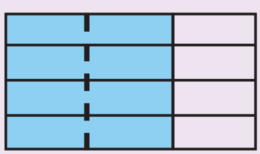
$\frac{2}{3}$

Fig.-a

$\frac{3}{4}$

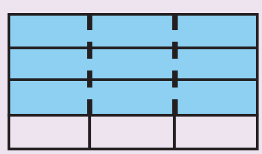
Fig.-b

- In the next step, draw the straight lines of Table-A in Table-B and draw the straight lines of Table-B in Table-A (Follow the picture below). You'll observe that the number of cells are equal in both the tables. For example: The number of the cells of the above table will become 12 (See the picture below). We can call the total number of cells as denominator and write this number in place of denominator in the fraction written above the table.
- Now, you count the number of cells in the parts you painted. Write counted number above. For example; in the following picture, Table A has 8 painted cells whereas Table B has 9 painted cells. These two numbers are the numerators of both the fractions. Now, write as per the following picture.



$\frac{8}{12}$

Fig.-a



$\frac{9}{12}$

Fig.-b

- The denominator or the divided parts of both the fractions are same. So, only by seeing the painted parts or numerators, it can be determined which fraction is greater. Here, 9>8, so, $\frac{9}{12} > \frac{8}{12}$.

- Practice a few more examples like this. Get your teacher's feedback on your work.

Tips: Make sure that you're drawing the rectangular cells or grids as per the instructions.

Now, try to answer the question asked above. First, write the part of the watermelon given to you by your friends in the blank space in fractions. Now, find out who got more and who got less watermelon among them and write the answers to the following questions.

Question 1: **In the above picture,** who got more share by comparing the watermelons received by friend-1 and friend-4??

Question 2: In the above picture, who got lesser share by comparing the watermelons received by friend-2 and friend-5?

Question 3: In the above picture, who got more share by comparing the watermelons received by friend-1 and friend-5?

Now observe an interesting thing, for question no.-1 and 2 you could easily find the answers, but in for question no.-3 you could not find the answer following the same rule, right? Think about the difference you got in question no.-3.

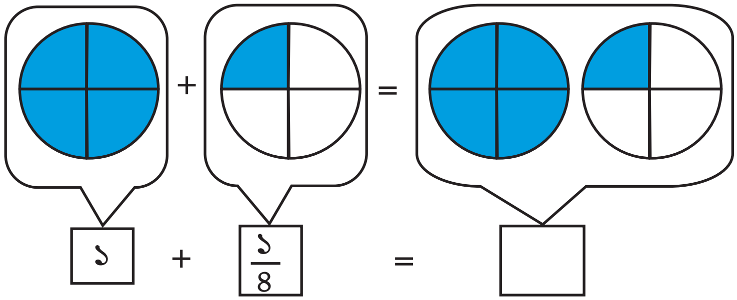
The difference is that the denominator of each fraction is different here. To make a comparison between them, the denominator of each fraction has to be converted to the same denominator. And for that, we need to find out the H.C.F of those two denominators. In previous class, you already have learned how to determine H.C.F.

For example, the denominators of the two fractions $\frac{2}{3}$ and $\frac{6}{10}$ are different. If we want to find out which of these two is bigger, then first we have to find out the H.C.F of 3 and 10. The H.C.F of 3 and 10 is 30. So, we need to make the denominators of both the fractions 30. To make the denominator of $\frac{2}{3}$ fraction to 30, its numerator and denominator has to be multiplied by 10. Then, $\frac{2}{3}$ will become $\frac{20}{30}$ Similarly, $\frac{6}{10}$ will be $\frac{18}{30}$. This time the comparison shows that $\frac{20}{30}$ fraction is larger between $\frac{20}{30}$ and $\frac{18}{30}$ fractions. So, between $\frac{2}{3}$ and $\frac{6}{10}$ fractions ; $\frac{2}{3}$ is the larger fraction.

Now, you have understood how to compare between two fractions if the denominators are different. Now solve the question-3 in the above table.

Improper Fraction and Mixed Fraction

Now, let's go back to Ratul. Ratul took 5 Pithas made by his mother to school for tiffin the next day.He will share the cake with his friends Mili, Harun and Tania during tiffin. Ratul started thinking how to divide these 5 Pithas among 4 people. Then, Tania said, there are 5 Pithas here and we will divide it among 4 people, we will take 1 Pithas each and then will divide the last Pitha into 4 parts and will take 1 part each from there. So, is it possible to find out, through addition, how much of Pithas each of them would get? You've learned the addition and subtraction of fractions in class five. Do the following addition accordingly and write the result in the blank space.



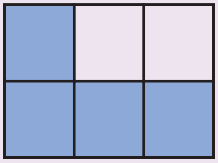
From the above discussion we could understand that Ratul and each of his friends will get $\frac{5}{4}$ parts of Pithas. Notice one thing here, the numerator of fraction $\frac{5}{4}$ is greater than the denominator. Fractions like this are known as Improper fractions. Again, we can break down the fraction $\frac{5}{4}$ as $1\frac{1}{4}$ where the fraction is written in coordination of an integer and a fraction. Thus, the fraction obtained by combining an integer and a fraction is called a .mixed fraction. This $1\frac{1}{4}$ fraction is thereby a mixed fraction. So, we understood that mixed fraction is not something different.

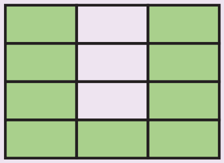
We can express improper fraction $\frac{5}{4}$ as mixed fraction $1\frac{1}{4}$. Now, let's see how to get improper fractions from mixed fractions.

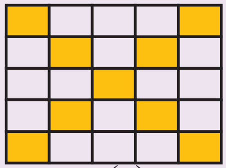
$$1\frac{2}{5} = \frac{5\times1+2}{5} = \frac{5+2}{5} = \frac{7}{5}$$

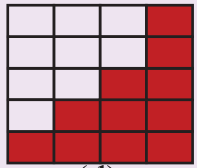
Individual Task: Solve the following problems in your exercise book and submit it to your teacher.

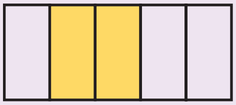
1. Express the following painted parts in fractions.

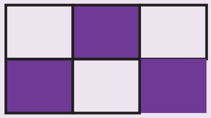

(a)

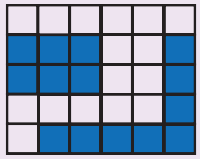

(b)


(c)

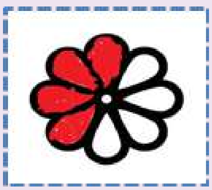

(d)

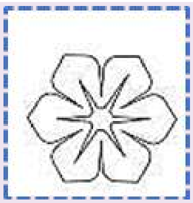

(e)

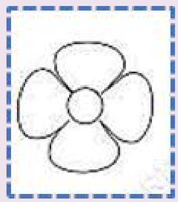

(f)

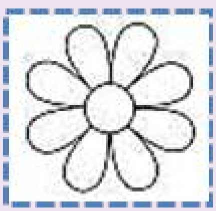

(g)

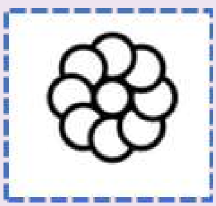
2. Paint specific parts of the pictures to express the fractions given beside the pictures. One is done for you.


$\frac{4}{8}$


$\frac{2}{6}$


$\frac{3}{4}$


$\frac{1}{4}$


$\frac{5}{8}$


$\frac{1}{2}$

3. Determine which pairs of fractions are larger and which are smaller from the 4 pairs of fractions given below.

$\frac{3}{10}$ and $\frac{2}{5}$	$\frac{5}{9}$ and $\frac{4}{7}$	$\frac{4}{9}$ and $\frac{2}{3}$	$\frac{7}{15}$ and $\frac{9}{12}$

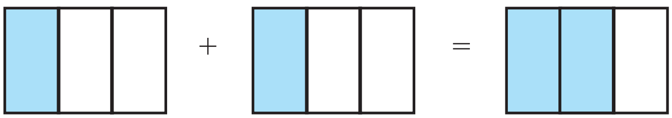
4. Draw the following mixed fractions on paper as grid and express them as improper fractions.

a) $2\frac{3}{7}$ b) $5\frac{5}{8}$ c) $3\frac{2}{5}$

Addition and Subtraction of fractions

Let's learn the techniques of addition and subtraction of fractions with the help of grid.

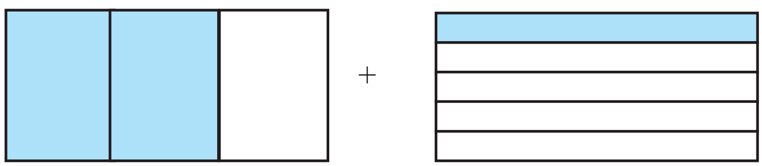
a)



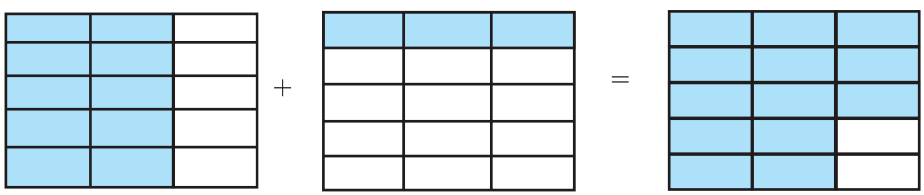
$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

b)

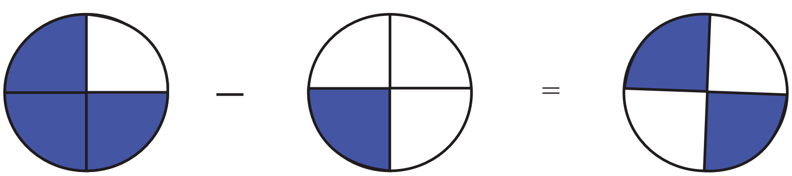
$\frac{2}{3}$ $\frac{1}{5}$



$$\frac{2\times5}{3\times5} = \frac{10}{15} \qquad \frac{1\times3}{5\times3} = \frac{3}{15} \qquad \frac{10}{15} + \frac{3}{15} = \frac{13}{15}$$

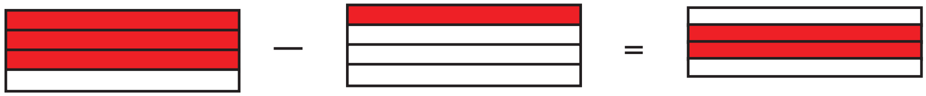


c)

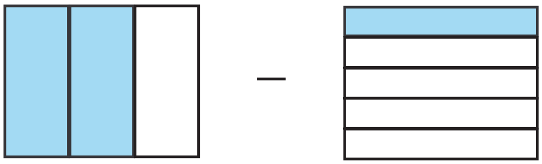


$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$$

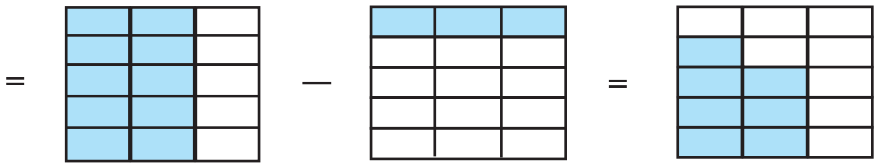
d) $\frac{3}{4}$ $-$ $\frac{1}{4}$ $=$ $\frac{2}{4}$



e) $\frac{2}{3}$ $-$ $\frac{1}{5}$



$\frac{2\times5}{3\times5} = \frac{10}{15}$ $\quad$ $\frac{1\times3}{5\times3} = \frac{3}{15}$ $\quad$ $\frac{10-3}{15} = \frac{7}{15}$



Individual worksheet: Complete the worksheet and submit it to your teacher on the next day.

Let's learn the techniques of addition and subtraction of fractions with the help of grid.

a) $\frac{1}{4} + \frac{1}{4}$ $\quad$ b) $\frac{1}{5} + \frac{2}{5}$ $\quad$ c) $\frac{4}{5} + \frac{6}{5}$ $\quad$ d) $\frac{3}{7} + \frac{1}{3}$

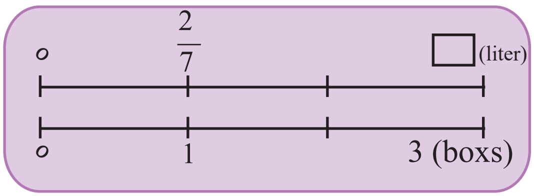
e) $\frac{5}{7} - \frac{2}{7}$ $\quad$ f) $\frac{1}{5} - \frac{1}{10}$ $\quad$ g) $\frac{4}{5} - \frac{2}{3}$ $\quad$ h) $\frac{5}{6} - \frac{1}{4}$

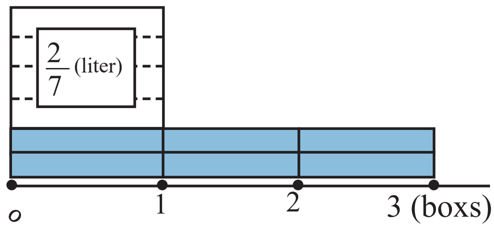
Multiplication of Fractions and Integers

It requires $\frac{2}{7}$ liters of milk to prepare one box of ice-cream. To prepare such three boxes of ice-cream, how much milk would be required?

We can use the following sentence to determine the total amount.

Milk required for one box of ice-cream	×	Total number of ice-cream boxes	=	Total amount of milk required





Here,

$$\frac{2}{7} = 2 \text{ units of } \frac{1}{7}$$

$$\frac{2}{7} \times 3 = (2 \times 3) \text{ units of } \frac{1}{7}$$

$$\frac{2}{7} \times 3 = \boxed{}$$

Let's calculate,

$$\frac{2}{7} \times 3 = \frac{2 \times 3}{7} = \frac{6}{7} \text{ Litre}$$

To multiply a fraction by an integer, we need to multiply the numerator by the integer keeping the denominator as it is.

$$\frac{A}{B} \times C = \frac{A \times C}{B}$$

Let's think how to calculate, $\frac{5}{12} \times 6$

Let’s compare and explain the following multiplications.

$$\frac{5}{12}\times 6 = \frac{5\times 6}{12} = \frac{\cancel{30}\,^{5}}{\cancel{12}\,_{2}} = \frac{5}{2}$$

$$\frac{5}{12}\times 6 = \frac{5\times \cancel{6}\,^{1}}{\cancel{12}\,_{2}} = \frac{5}{2}$$

Individual work: Solve the following problems by drawing grids in your exercise book and get it checked by your teacher.

Serial no	Problem	Solution
1.	It takes 1 deciliter of color to paint $\frac{7}{15}$ square meters of a board. How many square meters can be painted by 5 deciliters of color?	
2.	It takes $\frac{3}{8}$ kilogram sugar to prepare a bowl of Payesh. How many kilograms of sugar would be required to make such 16 bowls of Payesh?	
3.	Ask your parents, how many kilograms of rice your family needs every day? Then as per that amount, calculate the amount of rice required for your family in one month.	
4.	The weight of a 1 meter long metal tube is $\frac{5}{3}$ kg. What would be the weight of such a 6 meter long metal tube?	
5.	Find out how many students in your class like Math, how many students like English and how many students like both Math and English. Then determine, how much of the total students each information represents.	

Meaning of Multiplication:

Think about it, what does $\frac{2}{5} \times 3$ mean? How can we do this kind of multiplication? You must remember the method of multiplication by 'adding again and again'. Isn't it?

Well let's try to find out the meaning of $\frac{2}{5} \times 3$,

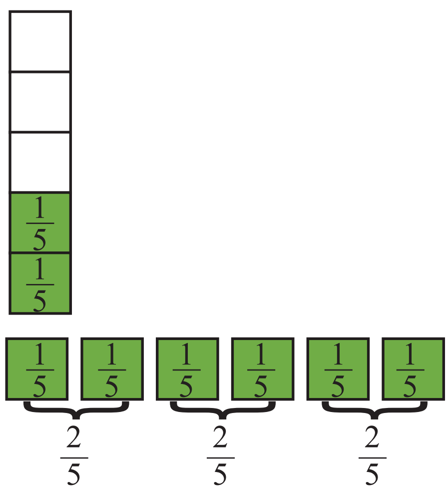
$\frac{2}{5} \times 3$ means taking $\frac{2}{5}$ for 3 times. That is, if we add $\frac{2}{5}$ for 3 times, we will get the product.

That is $\frac{2}{5}+\frac{2}{5}+\frac{2}{5} = \frac{2\times 3}{5} = \frac{6}{5}$

Now let's verify the problem in another way:

Let's try to solve this problem by using paper strips or round paper. You all must try to do this with strips of paper.

Take a strip, divide it into 5 equal parts then take 2 parts. So, this 2 parts will be equal to 2/5. Then make 3 bunches of $\frac{2}{5}$. [2 parts of $\frac{1}{5}$ will form 1 bunch of $\frac{2}{5}$, there will be 3 such bunches] If the strip of $\frac{1}{5}$ is used then the solution would look like the picture below.



Now, count the pieces, there are 6 pieces of $\frac{1}{5}$ in total or 3 bunches of $\frac{2}{5}$.

That is $\frac{2}{5} \times 3 = \frac{6}{5}$.

If we want, we can also write the product as below-

If we want, we can also write the product as below-

$\frac{2}{5}$, 2 units of $\frac{1}{5}$ So,

$\frac{2}{5} \times 3 = (2\times 3)$ units of $\frac{1}{5} = 6$ units of $\frac{1}{5} = \frac{6}{5}$ units.

So we can say, when multiplying an integer with a fraction, the product is obtained by multiplying the integer with the numerator of the fraction. The denominator remains unchanged.

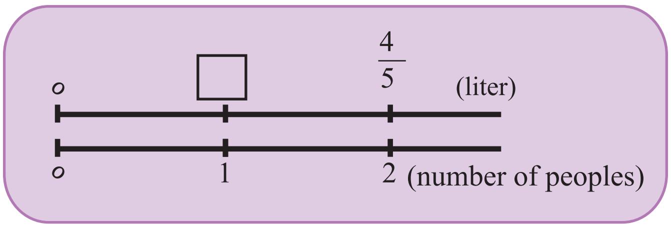
Individual task: Solve the following problems by drawing grids in your exercise book and get it checked by your teacher.

a) $\frac{2}{7} \times 7$ b) $\frac{3}{5} \times 15$ c) $\frac{7}{3} \times 9$ d) $\frac{5}{6} \times 8$ e) $3 \times \frac{2}{3}$

Division of Fractions and Integers

If $\frac{4}{5}$ liter of juice is divided equally between 2 people, how many liters will each of them get?

Total amount of juice	÷	Number of man	=	Amount of juice for 1 person



Express the problem through mathematical sentences:

Here, $\frac{4}{5} = 4$ units of $\frac{1}{5}$

So, $\frac{4}{5} \div 2 = (4 \div 2)$ units of $\frac{1}{5}$

$\frac{4}{5} \div 2 = \boxed{}$

Let's calculate, $\frac{4}{5} \div 2 = \frac{4 \div 2}{5} = \frac{2}{5}$

∴ Each of them will get ....liters of juice.

Now think about it, if we divide $\frac{4}{5}$ liters of juice equally among 3 people, then how to do it?

Mathematical sentence

$\frac{4}{5} \div 3$

$\frac{4}{5} \div 3 = \frac{4 \div 3}{5}$

but 4 can't be divided by 3 directly.

We can change the numerator for dividing the fraction by 3.

$\frac{4}{5} = \frac{4 \times 3}{5 \times 3}$

$$\frac{4}{5} \div 3$$
$$= \frac{4 \times 3}{5 \times 3} \div 3$$
$$= \frac{4 \times 3 \div 3}{5 \times 3}$$
$$= \frac{4}{5 \times 3}$$
$$= \frac{4}{15}$$

When dividing a fraction by an integer, the denominator has to be multiplied by that integer, keeping the numerator as it is.

$$\frac{A}{B} \div C = \frac{A}{B \times C}$$

I will express the fraction as least /smallest at the end of the calculation

I will express the fraction as least /smallest during the calculation

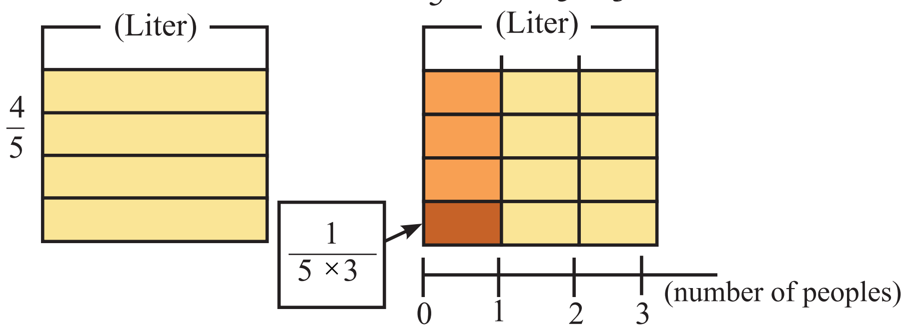
$$\frac{20}{9} \div 5 = \frac{20}{9 \times 5}$$
$$= \frac{\cancel{20}^{4}}{\cancel{45}_{9}}$$
$$= \frac{4}{9}$$

$$\frac{20}{9} \div 5 = \frac{\cancel{20}^{4}}{9 \times \cancel{5}_{1}}$$
$$= \frac{4}{9}$$

The calculation becomes easier if the fraction is expressed as the smallest/least form during the calculation.

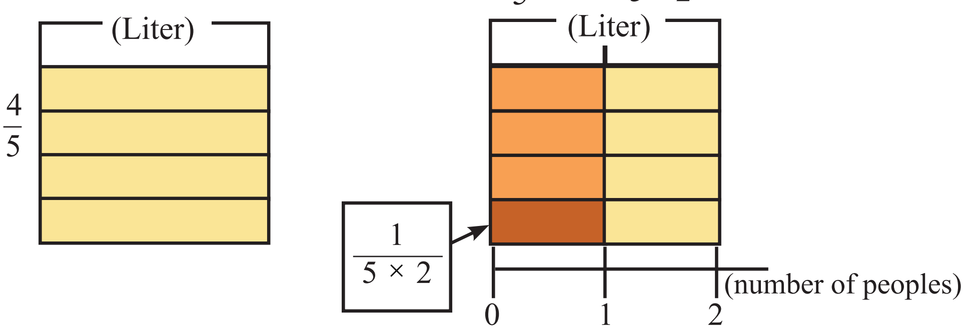
Now, let's think about the reasons why $\frac{4}{5} \div 3 = \frac{4}{5 \times 3}$ with the help of grid.

$$\frac{4}{5} \div 3 = \frac{4}{5 \times 3}$$



Now, let's try to determine $\frac{4}{5} \div 2$ with the help of grid following the same way.

$$\frac{4}{5} \div 2 = \frac{4}{5 \times 2}$$



Individual task: Solve the following problems by drawing grids in your exercise book and get it checked by your teacher.

Serial no	Problem	Solution
1	If $\frac{5}{6}$ liters of milk is divided equally among 5 people, how many liters will each person get?	
2	It takes 7/3 grams of sugar to make tea for everyone in your family. How many grams of sugar will be needed to make tea for only you?	
3	If 15/4 kg potatoes are divided equally among 5 people, how many kg will each of them get?	
4	It takes 2 deciliters of colour to paint 3/7 square meter walls. How many square meters of walls can be painted with 1 deciliter colour?	

Individual task: Solve by drawing grid.

a) $\frac{4}{3} \div 6$ b) $\frac{8}{7} \div 4$ c) $\frac{18}{11} \div 4$ d) $\frac{5}{2} \div 10$ e) $\frac{4}{9} \div 5$

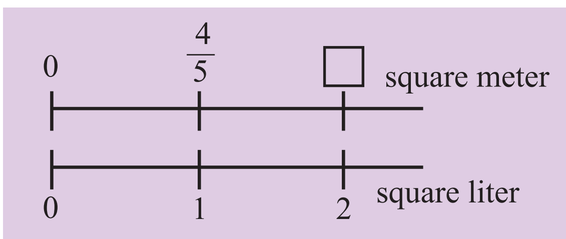
Pair work : On A4 paper or poster paper, mark 4/5 parts with a strip of paper. Find the quotient by dividing the marked part by 2. Create a few more similar problems and solve them using paper strips. Exchange the exercise book with your classmates to identify the mistakes and try to solve those through discussion. Take help from the teacher if needed.

Multiplication between Fractions

Think of a colour with 1 deciliter of which $\frac{4}{5}$ square meters can be painted.

(1) How many square meters can be painted by 2 deciliters of colour?

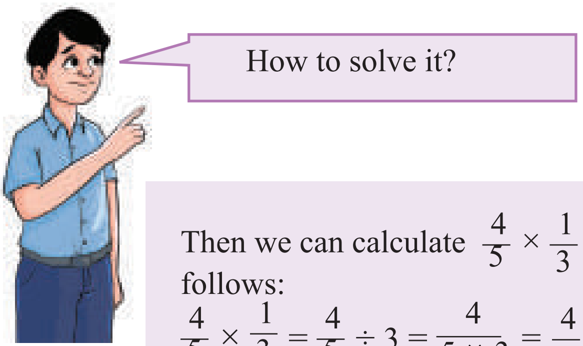
Area of painted part by 1 deciliter colour	×	Amount of colour	=	Total area of painted part



Let's calculate, $\frac{4}{5} \times 2 = \frac{8}{5}$ square meter

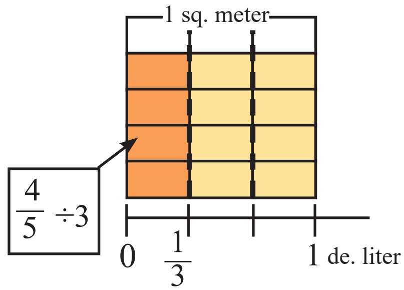
(2) How many square meters can be painted by $\frac{1}{3}$ deciliter color?

The number line shows that $\frac{4}{5} \times \frac{1}{3} = \frac{4}{5} \div 3.$



Then we can calculate $\frac{4}{5} \times \frac{1}{3}$ as follows:
$\frac{4}{5} \times \frac{1}{3} = \frac{4}{5} \div 3 = \frac{4}{5 \times 3} = \frac{4}{15}$

Now let's determine $\frac{4}{5} \div 3$ on a grid.

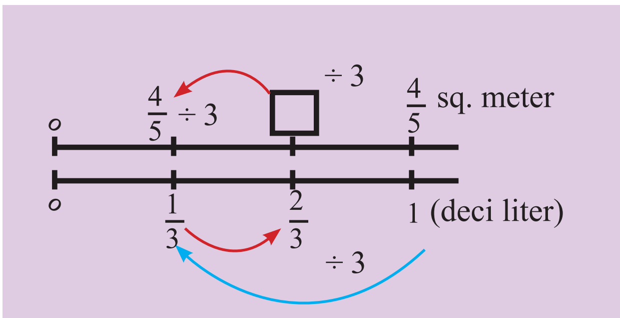


__________ sq. meter

3) How many square meters of area can be painted by $\frac{2}{3}$ deciliter color?

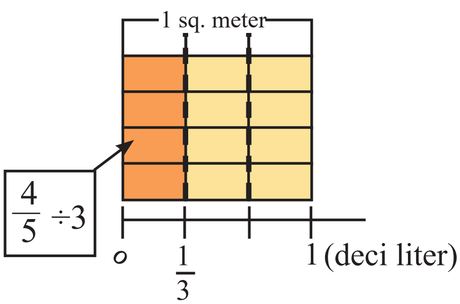
Here, the mathematical sentence: $\frac{4}{5} \times \frac{2}{3}$

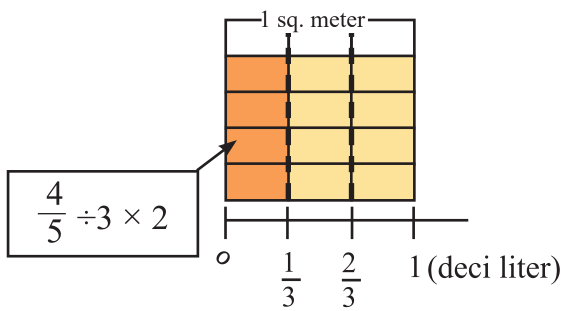
Let us first try to understand through the number line:



∴ Area of painted part by $\frac{2}{3}$ deciliter = Area of painted part by $2 \times \frac{1}{3}$ deciliter

Now, think with the help of the following grids:





Now, think with the help of the following grids:

$$\frac{4}{5} \times \frac{2}{3} = \left(\frac{4}{5} \div 3\right) \times 2$$

$$= \frac{4}{5 \times 3} \times 2$$

$$= \frac{4}{5} \times \frac{2}{3}$$

$$= \frac{8}{15}$$

Then we can calculate as follows:

$$\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$$

________________ sq. meter

When multiplying a fraction by a fraction, the numerator has to be multiplied by the numerator and the denominator by the denominator.

$$\frac{A}{B} \times \frac{C}{D} = \frac{A \times C}{B \times D}$$

$\therefore$ Let's think about how to calculate $\frac{3}{5} \times 2$ and $3 \times \frac{4}{7}$

We can turn an integer into a fraction containing 1 denominator and then can calculate.

$$\frac{3}{5} \times 2 = \frac{3}{5} \times \frac{2}{1}$$

$$= \frac{3 \times 2}{5 \times 1}$$

$$= \frac{6}{5}$$

$$2 \times \frac{4}{7} = \frac{2}{1} \times \frac{4}{7}$$

$$= \frac{2 \times 4}{1 \times 7}$$

$$= \frac{8}{7}$$

Surely, this is right: $\frac{3}{5} \times 2 = \frac{3 \times 2}{5} = \frac{6}{5}$

* How to calculate $2\frac{1}{3} \times 1\frac{2}{5}$?

We can calculate by turning mixed fractions into improper fractions.

$$2\frac{1}{3} \times 1\frac{2}{5} = \frac{\boxed{7}}{3} \times \frac{\boxed{7}}{5}$$

$$= \frac{49}{15} \text{ (or } 3\frac{4}{15}\text{)}$$

* Let's compare and explain how to calculate, $\frac{12}{25} \times \frac{5}{6}$

$$\frac{12}{25} \times \frac{5}{6} = \frac{60}{150}$$

$$= \frac{\cancel{60}^{\,2}}{\cancel{150}_{\,5}}$$

$$= \frac{2}{5}$$

$$\frac{12}{25} \times \frac{5}{6} = \frac{\cancel{12}^{\,2}}{\cancel{25}_{\,5}} \times \frac{\cancel{5}^{\,1}}{\cancel{6}_{\,1}}$$

$$= \frac{2}{5}$$

Again,

$$\frac{2}{5} \times \frac{10}{3} \times \frac{9}{8} = \frac{\cancel{2}^{\,1}}{\cancel{5}_{\,1}} \times \frac{\cancel{10}^{\,\cancel{2}}}{\cancel{3}_{\,1}} \times \frac{\cancel{9}^{\,3}}{\cancel{8}_{\,\cancel{4}\,2}} = \frac{3}{2} = 1\frac{1}{2}$$

Wow, even though it's a multiplication problem, we're not doing any multiplication; we're just expressing fractions in the smallest/least form.

Individual task: Solve with the help of grid and number line.

1. a) $\frac{4}{3} \times \frac{3}{4}$ b) $\frac{3}{5} \times \frac{10}{7}$ c) $\frac{5}{12} \times \frac{5}{10}$

   d) $\frac{7}{4} \times \frac{3}{5}$ e) $\frac{9}{8} \times \frac{3}{5} \times \frac{2}{27}$

2. Solve by drawing a grid in your exercise book and get it checked by your teacher by filling in the blanks.

Serial no	Fill in the blanks	Serial no	Fill in the blanks
1.	$\frac{2}{5} \times \frac{1}{3} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$	5.	$\frac{11}{13} \times \frac{21}{32} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$
2.	$\frac{5}{9} \times \frac{4}{9} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$	6.	$\frac{2}{5} \times \frac{\square}{\square} = \frac{2 \times \square}{5 \times \square} = \frac{8}{15}$
3.	$\frac{1}{6} \times 3\frac{1}{2}$ $= \frac{1}{6} \times \frac{\square}{2} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$	7.	$1\frac{2}{5} \times \frac{2}{17}$ $= \frac{\square}{\square} \times \frac{2}{17} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$
4.	$2\frac{2}{3} \times 3\frac{2}{3}$ $= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square \times \square}{\square \times \square}$ $= \frac{\square}{\square}$	8.	$= \frac{\square}{5} \times \frac{3}{\square} = \frac{\square \times \boxed{3}}{5 \times \square} = \frac{12}{35}$

Group work: Check the accuracy of the multiplication between fractions with the help of grid.

Material: Poster paper, A4 paper, Markers, Colour pencils.

* Fix groups as per the instructions of the teacher.

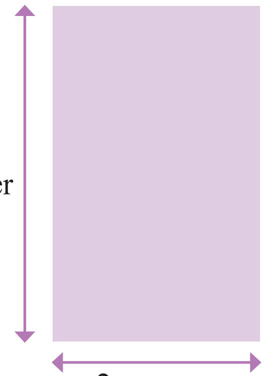
* Solve the following problem using the method of multiplication of fractions.

Let us determine the area of a rectangular board whose length is $\frac{6}{7}$ m and width is $\frac{2}{5}$ m.

Let us first remember the formula of determining the area:
Area of rectangular field= Length × Width

Express the problem in mathematical sentence:

$\frac{\square}{\square} \times \frac{\square}{\square} = \square$ Square meter.



$\frac{6}{7}$ meter

$\frac{2}{5}$ meter

Draw a grid using A4 paper to express fractions and observe the method of multiplication of two fractions.

Let's verify that the area of the rectangular board is $\frac{12}{35}$ sq.m with the help of the following diagram.

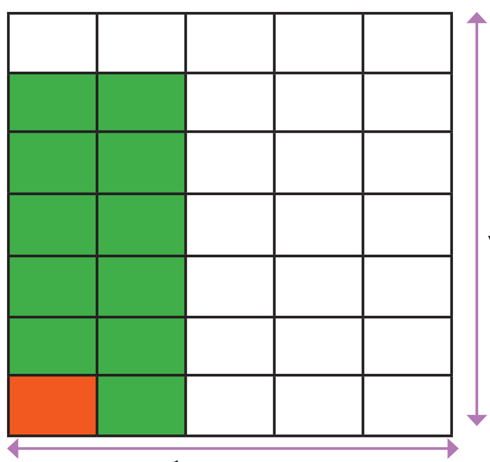
Here, $\frac{6}{7} \times \frac{2}{5} =$

$(6 \times 2) \times \left(\frac{1}{7\times5}\right)$ sq.m

So, it is $\frac{6\times2}{7\times5}$ sq.m

$\therefore$ area is $\frac{12}{35}$ sq.m

$=\left(\frac{1}{7\times5}\right)$ sq.m



1 meter

1 meter

- Discuss in groups, how the multiplication of fractions can be done with the help of grid and without even multiplying those fractions. If necessary, ask your queries to the teacher.
- Solve the mathematical problems given by the teacher and exchange the exercise bookswithin the group to check the accuracy.

Individual task: Solve the problems by drawing grids on A4 paper.

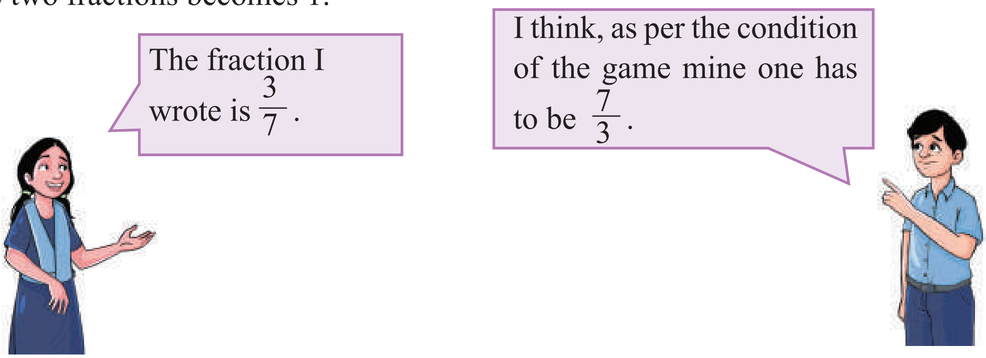
a) Let us determine the area of a rectangular wall with a length of $2\frac{3}{5}$ m and a width of $\frac{5}{6}$ m.

b) If the length of one side of a square garden is $3\frac{2}{3}$ m, then determine the area of the garden.

c) Check the solutions of mathematical problem a and b with the help of grid.

Reciprocal of Fraction

Riya and Ratul are playing an interesting game. Riya said to Ratul, “I will write a fraction in my exercise book. You also need to write such a fraction so that the product of the two fractions becomes 1.



Think like Ratul a little and say whether the fraction written by Ratul is correct or not? Okay, let’s calculate and see:

$$\frac{\cancel{3}}{\cancel{7}} \times \frac{\cancel{7}^{\,1}}{\cancel{3}} = 1$$

Why should the product of two fractions be 1?

The game has to be named. This game is called Reciprocal of Fraction game. Think, whether we can have any other name for the game or not. We can give another name to the game. The name is - Multiplicative Inverse game.

So we can say,

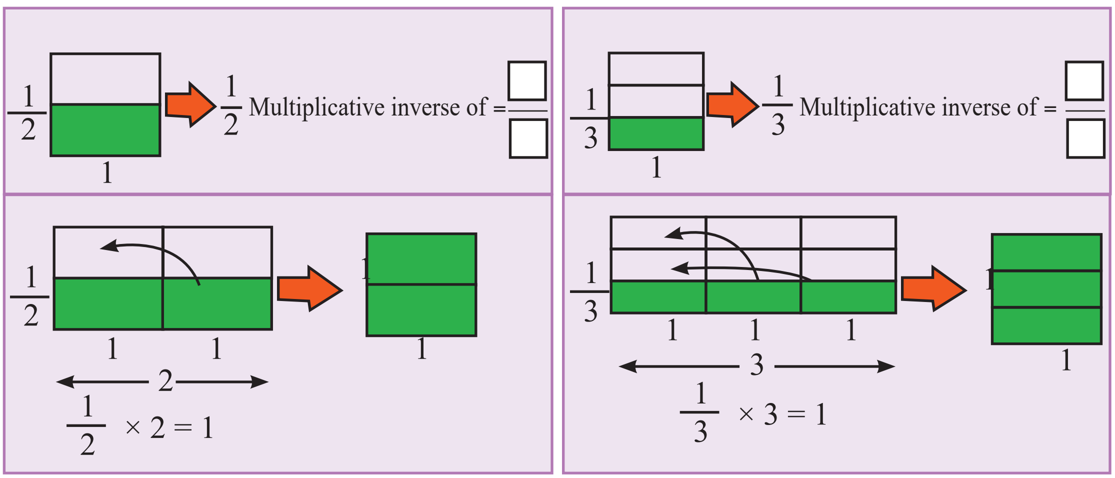
If the product of two non-zero fractions is 1, then any of the fractions is the reciprocal of fraction of the other or multiplicative inverse of the other.

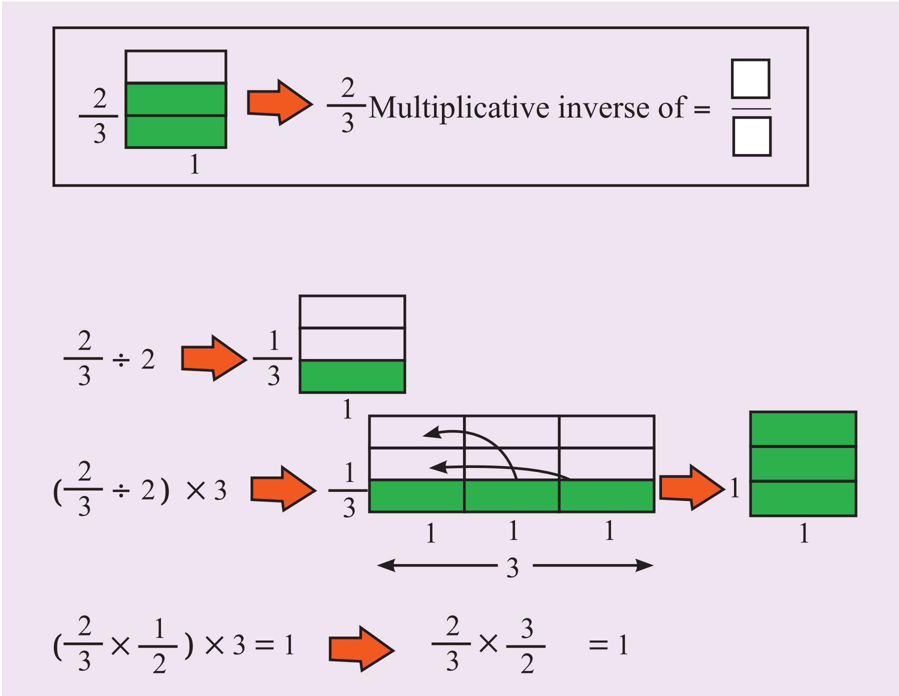
But keep in mind that the idea of 'Additive Inverse' that you got from the idea of negative numbers is totally different. If the sum of two fractions is zero (0), then they can be called 'positive inverse fraction' to each other.

Pair game: Just like Riya and Ratul, you can also play the game of reciprocal of fraction or multiplicative inverse game with your classmate by taking 10 fractions.

Multiplicative Inverse in Grid

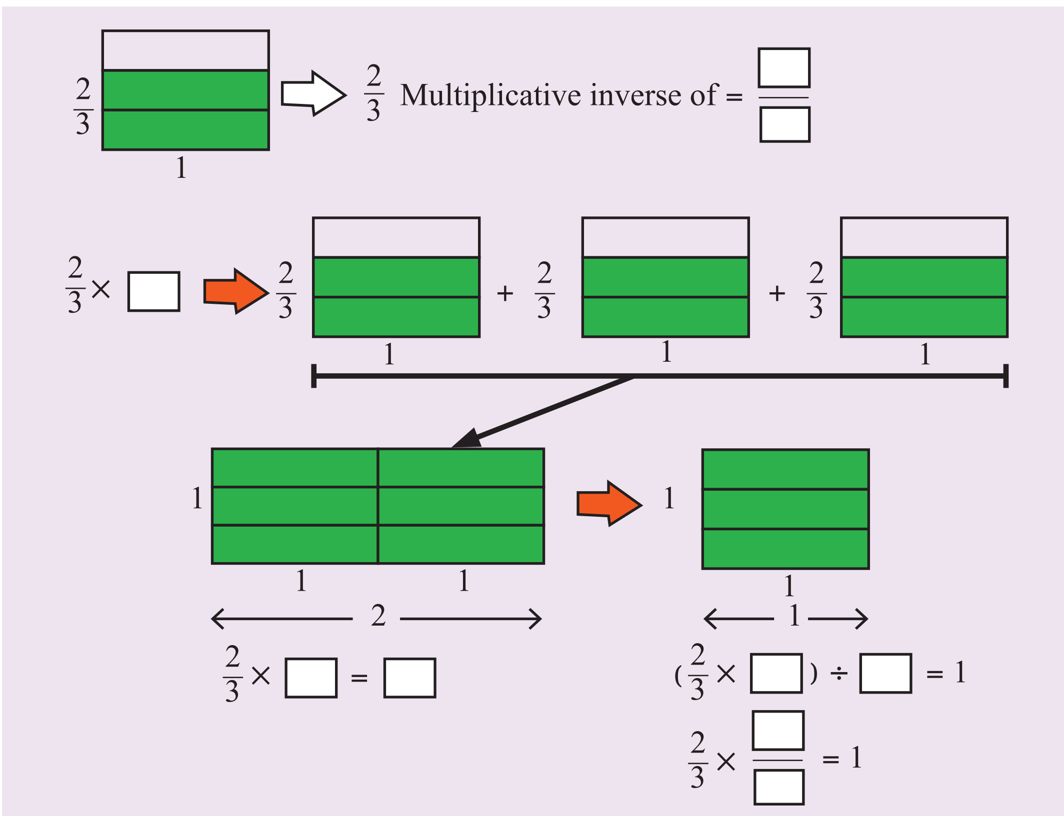
Let's determine multiplicative inverse of fractions by using the concept of determining the product of fractions.



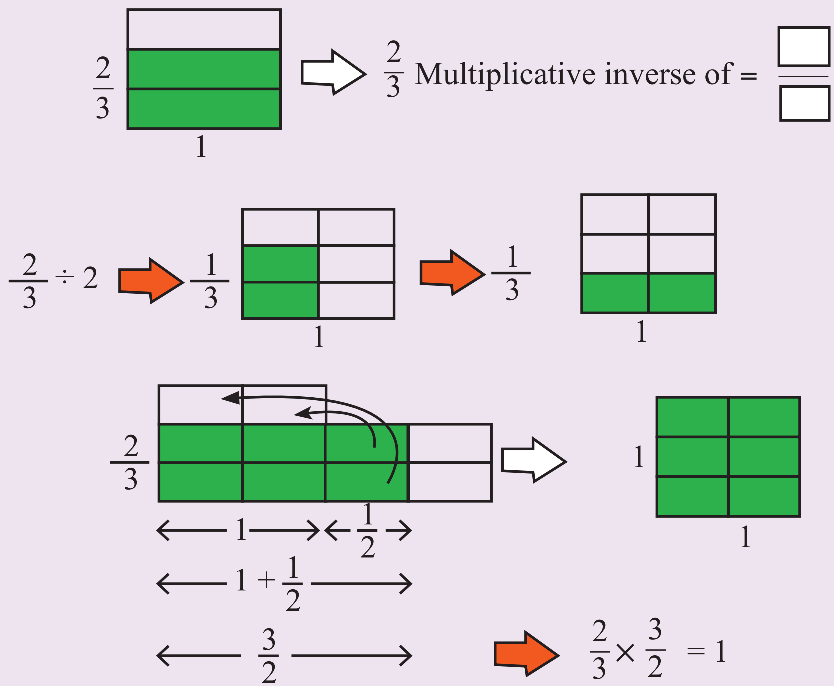


$\frac{2}{3}$
1
$\frac{2}{3}$ Multiplicative inverse of = $\frac{\square}{\square}$
$\frac{2}{3} \div 2$
$\frac{1}{3}$
1
$(\frac{2}{3} \div 2) \times 3$
$\frac{1}{3}$
1 1 1
3
1
1
$(\frac{2}{3} \times \frac{1}{2}) \times 3 = 1$
$\frac{2}{3} \times \frac{3}{2} = 1$

Again.



$\frac{2}{3}$
1
$\frac{2}{3}$ Multiplicative inverse of = $\frac{\square}{\square}$
$\frac{2}{3} \times \square$
$\frac{2}{3}$ + $\frac{2}{3}$ + $\frac{2}{3}$
1 1 1
1
1 1
2
1
1
1
$\frac{2}{3} \times \square = \square$
$(\frac{2}{3} \times \square) \div \square = 1$
$\frac{2}{3} \times \frac{\square}{\square} = 1$



Individual work: Determine the multiplicative inverse of the following fractions using grid.

a) 1 b) 5 c) $\frac{2}{5}$ d) $\frac{3}{7}$ e) $\frac{9}{7}$ f) $2\frac{3}{8}$

By analyzing Ratul and Riya's game and through the example of the grid we can reach the following conclusion–

The multiplicative inverse of a fraction can be obtained by interchanging the numerator and denominator of a fraction. $\Rightarrow$ Multiplicative inverse $\frac{A}{B} \to \frac{B}{A}$

Now try to find the answer to the two questions of Ratul and Riya. Determine the answers to the two questions with two or more examples and get it checked by the teacher.

How to determine the multiplicative inverse of integers?

What will be the additive inverse fraction of $\frac{8}{15}$?

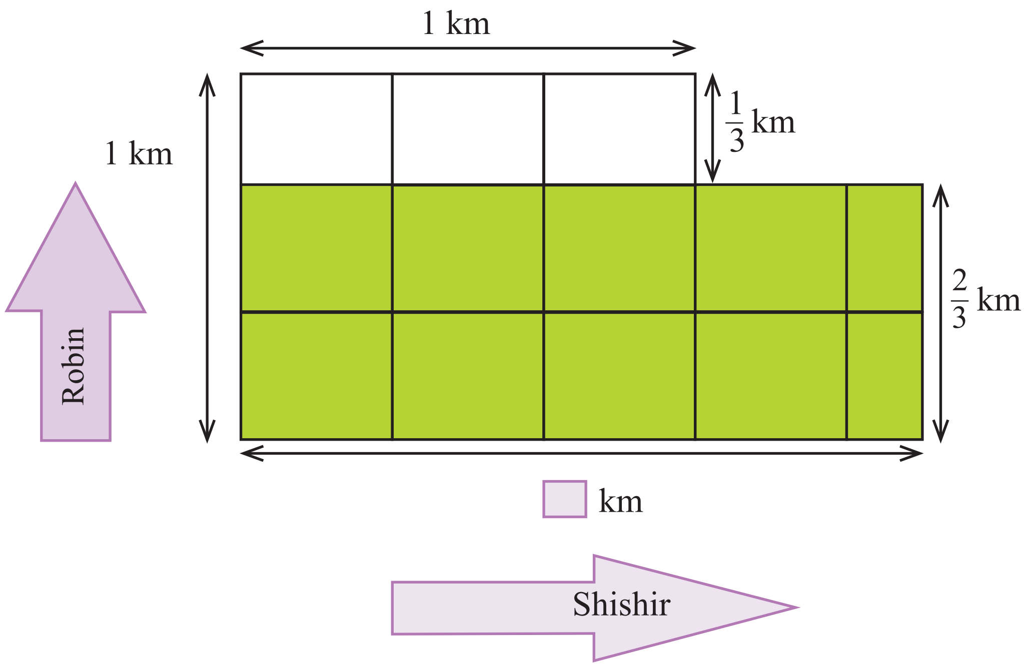
Individual task:

Serial no	Fraction	Reciprocal of fraction or multiplicative inverse	Serial no	Fraction	Reciprocal of fraction or multiplicative inverse
1.	$\frac{8}{5}$		6.	$\frac{1}{3}$	
2.	$\frac{3}{11}$		7.	$-\frac{2}{3}$	
3.	$\frac{4}{7}$		8.	$\frac{0}{5}$	
4.	$\frac{8}{17}$		9.	8	
5.	2		10.	-12	

Field of Magic

There is a huge magic field just besides Robin's house. The length and width of the field changes every morning but the shape of the field remains rectangular and the area also remains unchanged. So, one day Robin walked around and measured the length and width of the field and found both as 1 kilometer. That day Robin knew it for sure that the shape of the field was square. So, the area of the field = 1 sq. km. And since the area of the field doesn't change, then the area of the field remains 1 square kilometer every day. The next day Robin went to the magic field and saw that the width of the field had decreased to $\frac{2}{3}$ meter.

Now he began to think how long the length could be now? Definitely, it's more than 1 km now. But Robin doesn't feel like walking so far along the length. If you want to help Robin, then determine, what was the length of the field that day?



Robin then came up with a simple idea to know both length and width. He used to go to the magic field every day with his friend Shishir. To measure the length and width of the field, Robin and Shishir would start walking at the same speed along both sides of the field, as shown in the picture. Whenever one of them reached the far end of the field, that is, the length or width, he would shout loudly and tell his friend to stop. Then the other friend did not have to walk the full distance. Whenever a distance of length or width was found, they would determine the other distance from there. See if you can come up with the same idea in the case of the events of the following days.

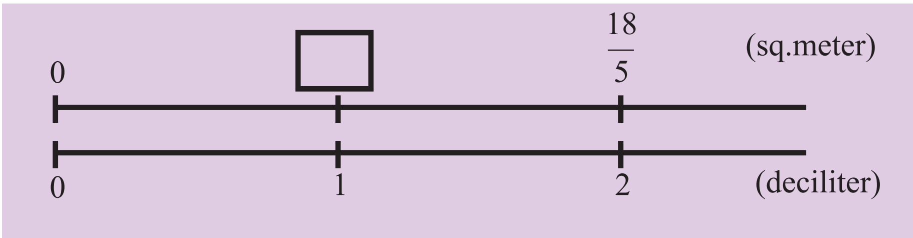
Individual task : Now think about it, if the length of the magic field is too big, what will be the width? Again, if the width is much smaller and close to zero, what will be the value of length?

Day	Distance of the field to the direction Robin was walking (Length or width)	Distance of the field to the direction Shishir was walking (Length or width)	Who has covered the full distance and why
01/01/2022	$\frac{1}{2}$	2	Robin because $\frac{1}{2} < 2$
02/01/2022	3	$\frac{1}{3}$	Shishir because $\frac{1}{3} < 3$
03/01/2022	$\frac{1}{4}$	$\square$	$\square$ because $\square$
04/01/2022	$\frac{2}{5}$	$\square$	$\square$ because $\square$
05/01/2022	$\square$	$\frac{1}{10}$	$\square$ because $\square$
06/01/2022	$\frac{1}{10000}$	$\square$	$\square$ because $\square$
07/01/2022	10000	$\square$	$\square$ because $\square$
08/01/2022	$\frac{7}{3}$	$\square$	$\square$ because $\square$
09/01/2022	$\square$	$5\frac{2}{3} = \frac{\square}{\square}$	$\square$ because $\square$

Division between Fractions

Blue colour was bought to paint the walls of your house. With 2 deciliter of the colour $\frac{18}{5}$ sq.m wall can be painted. How much of that wall can be painted with 1 deciliter of colour?

Let us first express the problem in mathematical sentences with the help of number line:



Total area of painted wall ÷ Amount of colour = Area of painted wall by 1 deciliter colour

Let's calculate, $\frac{18}{5} \div 2 = \frac{18}{5\times2} = \frac{\square}{\square}$

................Square meter

Let's solve another problem.

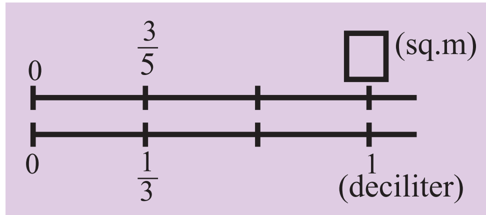
It takes $\frac{1}{3}$ deciliter colour to paint $\frac{3}{5}$ sq.m walls. How many square meters of that wall can be painted with 1 deciliter of colour?

Total area of painted wall ÷ Amount of colour = Area of painted wall by 1 deciliter colour

How can we calculate this?

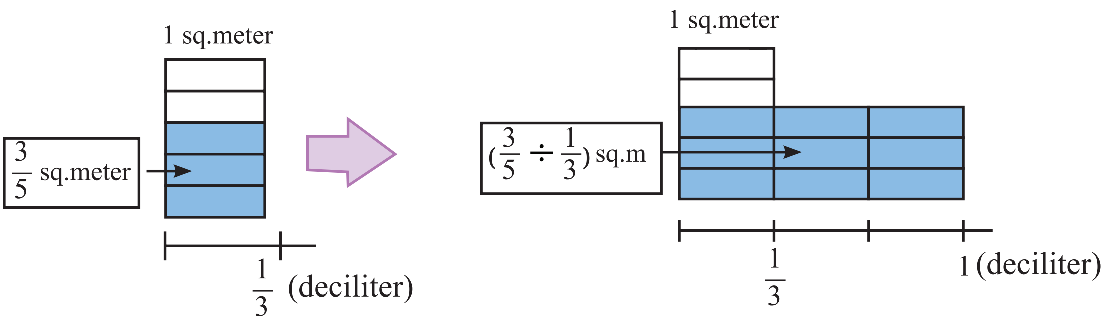
Mathematical Statement : $\frac{3}{5} \div \frac{1}{3}$ $\boxed{}$

We can think of it through the following design:



Let's think how to calculate, $\frac{3}{5} \div \frac{1}{3}$ using diagrams.

We will determine the area which 1 deciliter of colour can paint.



∴ 1 deciliter is 3 times of $\frac{1}{3}$ deciliter.

∴ 1 deciliter colour can paint 3 times more of the parts than $\frac{1}{3}$ deciliter colour can paint.

$$\frac{3}{5} \div \frac{1}{3} = \frac{3}{5} \times \frac{3}{1} = \frac{3 \times 3}{5} = \boxed{}$$

______ (sq.meter)

Now let's try to solve the following problem with the help of the above two solving methods.

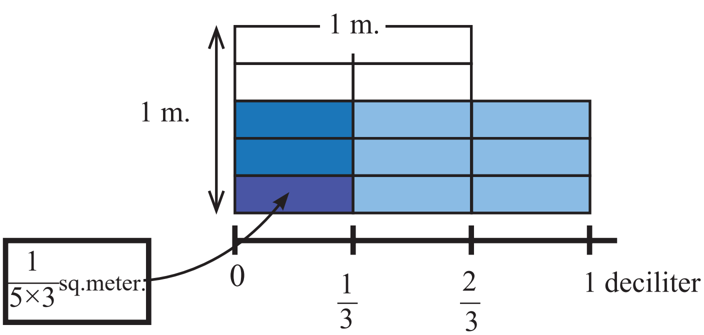
Solve the following problem

It takes $\frac{2}{3}$ deciliter colour to paint $\frac{3}{5}$ square meter of wall. How many square meters of that wall can be painted with 1 deciliter of colour?

Let us check the accuracy of the division of fractions in a few more ways.

Counting the pieces/ parts of shape with the help of grid.

In grid 1 square meter area is divided into 5×3 rectangular shape.



There are (3×3) pieces of size.

As a result, each rectangular part, its area = $\frac{1}{5\times 3}$ square m.

Now, 3 × 3 of sized pieces can be painted by 1 deciliter of color.

So, 1 deciliter can paint:

$$\frac{3}{5} \div \frac{2}{3} = (3 \times 3) \times \frac{1}{5 \times 3} = \frac{3 \times 3}{5 \times 2} = \frac{\square}{\square}$$

Through Reciprocal of Fraction:

The quotient doesn't change if the divisible and the divisor is multiplied or divided by the same number.

For example : 6÷2= 3 so, (6×5) ÷(2×5)=30÷10=3

Again, (6÷2)÷ (2÷2)=3÷1=3

In case of fractions, we multiply or divide the numerator and the denominator by the same number in order to get equivalent fraction. We can use the same concept in regard to division of two fractions.

$$\frac{3}{5} \div \frac{2}{3} = \left(\frac{3}{5} \times \frac{3}{2}\right) \div \left(\frac{2}{3} \times \frac{3}{2}\right)$$

$$= \left(\frac{3}{5} \times \frac{3}{2}\right) \div 1 = \frac{3}{5} \times \frac{3}{2}$$

$$= \frac{3 \times 3}{5 \times 2}$$

$$= \frac{\square}{\square}$$

The concept of multiplication and division of fractions with integers and through reciprocal of fractions, two fractions can be divided into several more ways.One such way is shown below:

$$\frac{4}{9} \div \frac{2}{3} = \left(\frac{(4 \div 2) \times 2}{(9 \div 3) \times 3}\right) \div \frac{2}{3}$$

$$= \left(\frac{(4 \div 2)}{(9 \div 3)} \times \frac{3}{3}\right) \div \frac{2}{3}$$

$$= \frac{(4 \div 2)}{(9 \div 3)} \times \frac{2}{3} \div \frac{2}{3}$$

$$= \frac{(4 \div 2)}{(9 \div 3)} \times \left(\frac{2}{3} \div \frac{2}{3}\right)$$

$$= \frac{(4 \div 2)}{(9 \div 3)} \times 1 = \frac{(4 \div 2)}{(9 \div 3)} = \frac{\square}{\square}$$

So, according to all the above methods we can say:

In case of division of fractions, we first multiply the fraction with the reciprocal of fraction.

$$\frac{A}{B} \div \frac{C}{D}$$

$$\frac{A}{B} \times \frac{D}{C} = \frac{A \times D}{B \times C}$$

Now, think how to calculate, $1\frac{4}{7} \div 2\frac{5}{14}$

At first, let's express the two fractions as improper fractions.

$$1\frac{4}{7} \div 2\frac{5}{14}$$

$$= \frac{\square}{\square} \div \frac{\square}{\square}$$

$$= \frac{\square}{\square} \times \frac{\square}{\square}$$

$$= \frac{\square}{\square}$$

Now, let's think how to calculate, $\frac{5}{8} \div \frac{15}{32} \times \frac{1}{12}$.

At first, let's multiply $\frac{5}{8}$ with the reciprocal fraction of $\frac{15}{32}$.

$$\frac{5}{8} \div \frac{15}{32} \times \frac{1}{12} = \frac{5}{8} \times \frac{\square}{\square} \times \frac{1}{12} = \frac{\square}{\square}$$

Individual task: Calculate by drawing the grid, fill up the table and show it to the teacher.

Serial no	Fill up the table	Serial no	Fill up the table
1.	$\frac{8}{9} \div \frac{2}{3} = \frac{8 \div 2}{9 \div 3} = \frac{\square}{\square}$	5.	$\frac{11}{13} \div \frac{11}{13} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$
2.	$\frac{12}{25} \div 1\frac{1}{5}$ $= \frac{\square}{\square} \div \frac{\square}{\square}$ $= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$	6.	$3\frac{1}{5} \div \frac{2}{5}$ $= \frac{\square}{\square} \div \frac{\square}{\square}$ $= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$
3.	$\frac{8}{5} \div \frac{4}{15} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$	7.	$\frac{20}{45} \div \frac{\square}{\square} = \frac{20 \div}{45 \div} \frac{\square}{\square} = \frac{4}{5}$
4.	$\frac{32}{12} \div 2\frac{2}{3} = \frac{\square}{\square} \div \frac{\square}{\square}$ $= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$	8.	$\frac{\square}{\square} \div \frac{2}{45} = \frac{\square}{\square} \times \frac{25}{2}$ $= \frac{\square}{\square} \frac{\times 25}{\times 2} = \frac{3}{4}$

Now read the story of the flute player of Subarnapur and find out how the prize of the shepherd boy can be divided.

The flute player of Subarnapur

There was a shepherd named Bashir in Subarnapur village. Bashir goes to the field with a herd of cows very early in the morning and returns home before evening. However, the villagers know Bashir as the flutist shepherd. Since, Bashir used to play flute in his leisure. The melody of his flute was strangely charming. At noon when the cows graze in the field on their own at that time Bashir sits under the shadow of a tree and brings out his flutes from his bag. As soon as the flute is played, all the magical tunes start coming out, and if someone passes by at that time then he is forced to stand because of the wonderful melody. One day the king of Subarnapur was passing by that field. The time was around noon, Bashir's herd of cattle was grazing in the field and Bashir was playing the fluteas usual. The king became astonished hearing the melody of the flute, he has never heard such wonderful melody ever before! All at once, he sent his Uzir to inquire who in his kingdom plays flute such melodiously. Uzir came before the king with Bashir.Bashir was too scared that he was brought before the king, he could not think of what he had done wrong. The king then removed the fear of Bashir by admiring his flute playing skills, and also invited him to the royal court to play the flute the next day in front of everyone. Afterwards, the king asked for Bashir's leave and said goodbye.

- Bashir was very happy as he had never been to the royal court before.But soon he fell into tension. Ashe has no good clothes, no shoes to go to the royal court, not even a car / vehicle to travel so far!Bashir hurriedly returned to his house with the herd of the cows from the field. He then shared everything with his neighbours and asked for their help.
- An old woman came forward.She said, 'I will make you a beautiful dress.But in return you have to give me one tenth of the reward you will get.' Bashir calculated in his mind, "If I get 50 gold coins, I have to give $\square$ to the old woman." Bashir agreed to the old woman's proposal.
- Then a shoemaker came forward. He said, 'I'll make you a shoe. But in return you should pay two-tenths of what reward you will get.'Bashir calculated in his mind, 'If I get 50 gold coins, I have to give $\square$ to the shoemaker.' Bashir also agreed to the shoemaker's offer.
- Finally, a blacksmith came forward. He said, 'I will make you a very strong vehicle. But in return you have to give me one-fifth of the reward you will get.'Bashir calculated in his mind,' If I get 50 gold coins, I have to give $\square$ to the blacksmith.'Bashir also agreed to the blacksmith's offer.
- On the next day Bashir went to the king's court with his new clothes, shoes and vehicle. After taking the king's permission, he played the flute. Everyone in the royal court became very pleased. The king was also very pleased and gave Bashir 100 gold coins. Bashir became very happy to receive this gift.

Now, answer the questions:

a. How many gold coins will the old woman get?

b. How many gold coins will the cobbler get?

c. How many gold coins will the blacksmith get?

d. How many gold coins will remain with Bashir's finally?

Group work : First, all the members of the group should read the story.

'The old woman of Achinpur and her herd of goats'

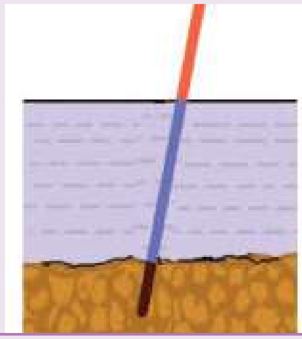
An old woman lived in a village called Achinpur. There was no one in her three clans. There were only 3 girls and 19 goats. The old woman decided one day that she will divide all the goats among her daughters. The old woman said,

- The elder daughter will get $\frac{1}{2}$ of my goats,
- The second daughter will get $\frac{1}{4}$ of my goats,
- The youngest daughter will get $\frac{1}{5}$ of my goats.

The girls got stunned hearing this. 19 goats cannot be divided into 2 parts, or 4 parts, or 5 parts! How will they divide the goats now? The three daughters of the old woman were not getting any idea to divide the goats as per the condition given by the old woman. At that time, a small boy from the neighborhood was taking his goat from there. Seeing the tensed daughters of the old woman, the boy asked about the reason. The little boy heard everything and said it is not a problem at all. You take my goat and then the total number of goats is 20. Now, divide the goats as your mother wants. But don't forget to return my goat once you finish dividing the goats.

Now you discuss in groups and decide how the old woman will divide the goats among her daughters. Act out this story in the classroom with all the members of the group.

Individual work : Solve the problems in the table below by drawing a grid on A4 paper.

Serial no	Problem	Solution
1.	$\frac{1}{6}$ part of a bamboo is on the ground, $\frac{1}{4}$ part is inside water and the rest is above water. If the length of the upper part of the water is $1\frac{1}{4}$ meters, how many meters of bamboo is inside the water? 	
2.	The area of a garden is 30 sq.m . Fruits have been cultivated in $\frac{3}{5}$ of this garden and flowers in $\frac{1}{10}$ part. Determine the area of the cultivated part.	
3.	Mr. Mokbul kept $\frac{1}{5}$ of his property for himself and divided the remaining property equally between his two children. a) How much of the total property did each children get? b) If Mr. Mokbul's own share is worth Taka 2,00,000, how much did each child get in monetary value?	a) b)
4.	Record the time you take from your home to school for 5 days. Then calculate your average speed in hours for one day.	

Game of Decimal Place Value

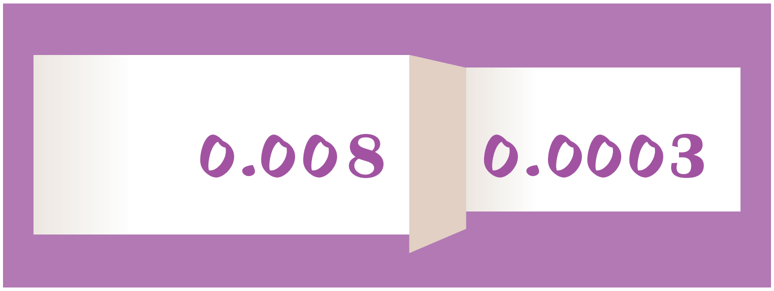
You've learned about the decimal place value in your previous class. In here, you'll be able to determine the decimal place value much easily through a game. Play the game with your classmates following the instructions given below and with the assistance of your teacher. You can also try this game at home.

Steps of the game

- Make a pair with your classmate as per the teacher's instruction.
- At first, divide a white A4 size paper into four parts as shown in the picture below. Then you will start the game with a piece from those four parts.

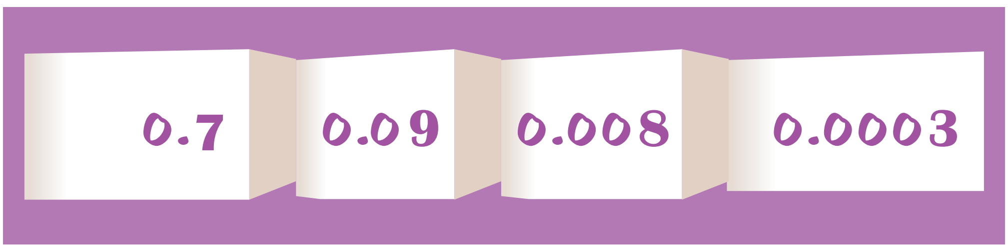
- Take a look at how to make numbers by folding the paper as shown in the picture. You will create secret numbers from tenths to thousands following this way. For example: How to make the number 0.7983 is shown in the picture-
- At first, you have to write the number 0.0003 on the far right side of the paper.

- Fold the paper from the edge of the "0" on the left side and cover the three "0" after the decimal point so that only the "3" is visible.
- Then write the number 0.008 on the paper.



- Then fold the paper in the same way and write the number to make the number 0.7983 at the end like the following figure on the paper.

- Show your folded paper to your teacher. Like in the above picture 0.7938 is seen. Again, open the folded paper to observe the place value of each number. For example: The figure below shows the place value of each digit of the number 0.7938.

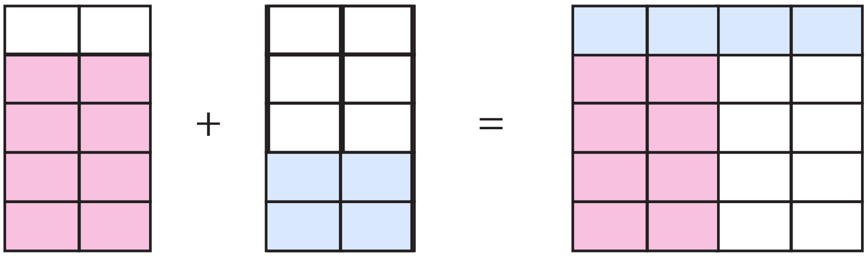
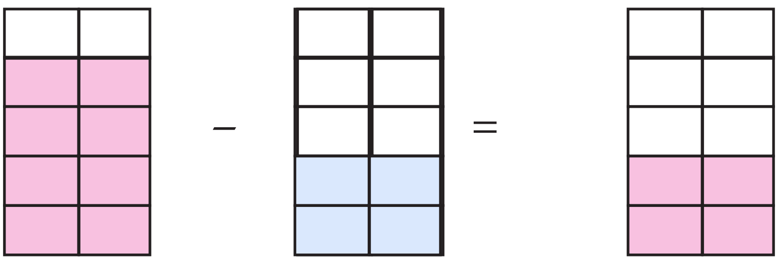


- You'll preserve and verify your work. Finally, the teacher will verify the accuracy of your work.
- Every time you make a number, you must write the number in words and numbers in your exercise book.

Individual task: You learned about decimal numbers in the previous class. Let's try to remember the idea of decimal numbers by filling up the table below.

Name of place						Number
Hundreds (100)	Tens (10)	Ones (1)	Decimal (0.1)	Percentage (0.01)	Thousands (0.001)	
3	1	2	4	7	2	312.472
5	3	7	9	1	4	
0		5		4	3	85.143
7	2			5		721.654
						620.801

The relationship between addition and subtraction of decimal fractions and of common fractions

<table>
<tr><th>Addition and subtraction of fractions</th><th>Addition and subtraction of decimal fractions</th></tr>
<tr><td>$\frac{4}{5} + \frac{2}{5} = \frac{6}{5}$</td><td>0.8 + 0.4 = 1.2</td></tr>
<tr><td colspan="2">
$\frac{4}{5} = \frac{8}{10} = 0.8$ $\frac{2}{5} = \frac{4}{10} = 0.4$ $\frac{6}{5} = \frac{12}{10} = 1.2$</td></tr>
<tr><td>$\frac{4}{5} - \frac{2}{5}$</td><td>0.8 - 0.4 =</td></tr>
<tr><td colspan="2">
$\frac{4}{5} = \frac{8}{10} = 0.8$ $\frac{2}{5} = \frac{4}{10} = 0.4$ $\frac{2}{5} = \frac{4}{10} = 0.4$</td></tr>
</table>

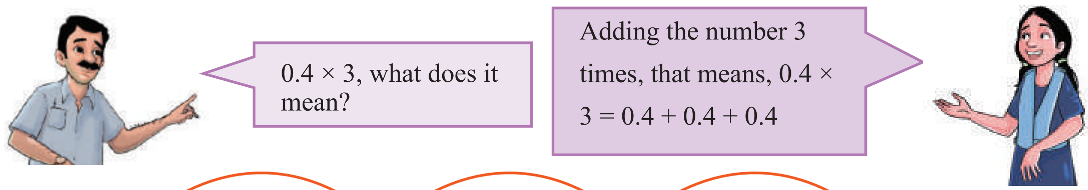
Individual task : Complete the table below with pictures

Addition of fractions		Addition of decimals	
$\frac{4}{5} + \frac{3}{10} =$		? + ? =	
$\frac{4}{5} - \frac{3}{10} =$		? - ? =	

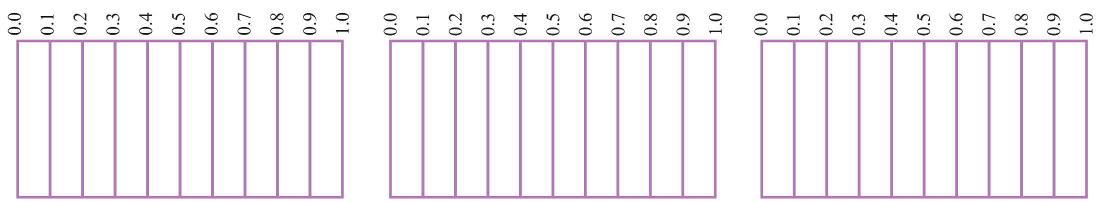
Multiplication of decimal fractions and integers

$0.4 \times 3 = ?$

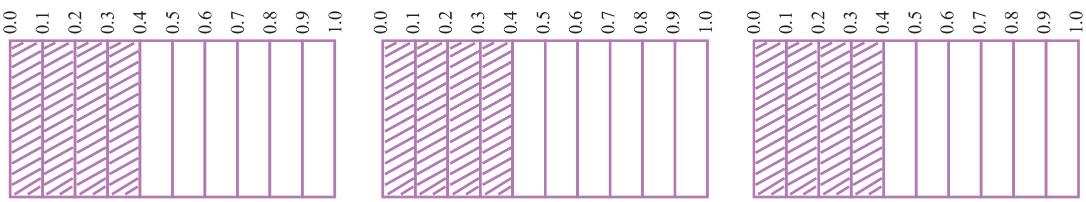
Solve through number lines and grids.



As per the instructions of the teacher draw and write the product in your exercise book.
Now solve the above mathematical problem through grid or rectangular cell.
Now, each of you have to draw three grids, each of the grids must have ten cells, in your exercise book.



Now fill $0.4 = \frac{4}{10}$ parts from each grid using color pencil.



Now, take 0.4 for 3 times to determine 0.4×3.
Then, calculate through the grid and see what the product is if you take 0.4 for 3 times?
Let's see how the product is calculated using the grid:

$$0.4 \times 3 = \frac{4}{10} \times 3 = (4 \times 3) \times \frac{1}{10} = 12 \times \frac{1}{10} = \frac{12}{10} = 1.2$$

From the method of multiplying by grid we found an easy way to do this multiplication of $0.4 \times 3 = 1.2$.

Numbers need to be multiplied like normal multiplication without thinking about decimal points. For example: $4 \times 3 = 12$.

We need to place decimal points (calculating digits) in the product following the decimal point's place in the multiplicand. That means, $0.4 \times 3 = 1.2$.

And this is the traditional method of multiplying decimal fractions by integers.

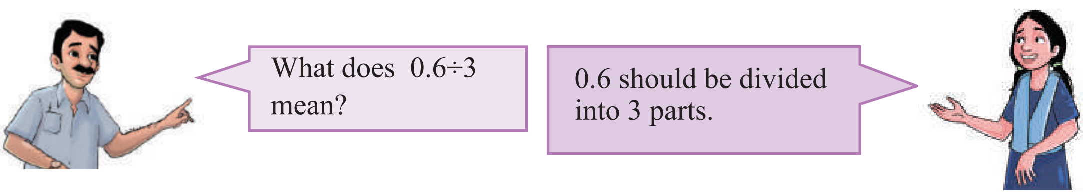
Individual work : Solve by drawing grids.

a) 0.4 x 5 **b) 0.7 x 9** **c) 0.2 x 13** **d) 0.72 x 6** **e) 0.27 × 3**

Division of decimal fractions and integers

$0.6 \div 3 = ?$

Solve through number lines and grids.



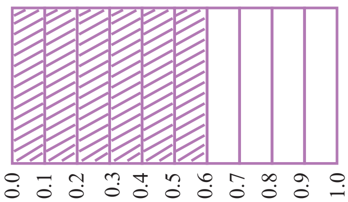
Picture-1

Picture-2

Draw and write the quotient in your exercise book as per the instructions of the teacher. Now solve the above mathematical problem through grid or rectangular cell.

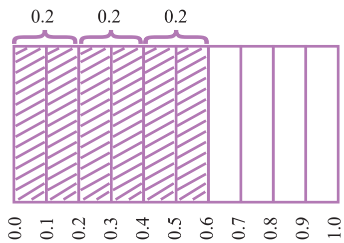
Draw a grid in each of yours' exercise book and divide the grid into ten equal parts.

Now mark $0.6 = \frac{6}{10}$ part from the grid.



Now, to find $0.6 \div 3$, divide 0.6 into three parts and see how many parts are there in each divided part.

After dividing in grids, it will look like the following;



Then, calculate through the grid and see what the quotient is if 0.6 is divided by 3?

Here is how to find the quotient with the help of grid:

$$0.6 \div 3 = \frac{6}{10} \div 3 = (6 \div 3) \times \frac{1}{10} = 2 \times \frac{1}{10} = \frac{2}{10} = 0.2$$

From the method of dividing by grid we found an easy way to divide $0.6 \div 3 = 0.2$

The numbers have to be divided like normal divisions without thinking of decimal points.

For example: $6 \div 3 = 2$

The decimal point should be placed at the point where the divisor has a decimal point (or as many cells as there are after the decimal of the divisor) that means, $0.6 \div 3 = 0.2$

And this is the traditional method of dividing decimal fractions by integers.

Individual task : Solve through grid.

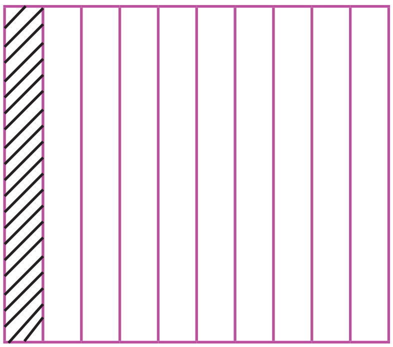
a) $7.5 \div 5$ b) $9.6 \div 8$ c) $1.4 \div 7$ d) $1.05 \div 5$ e) $0.09 \div 3$

Multiplication between decimals

Let's figure out - how to multiply a decimal fraction with another decimal fraction? Is it like the multiplication of decimal fractions with integers or in some other way? Let us think of a solution to the following mathematical problem.

$0.2 \times 0.3 = ?$

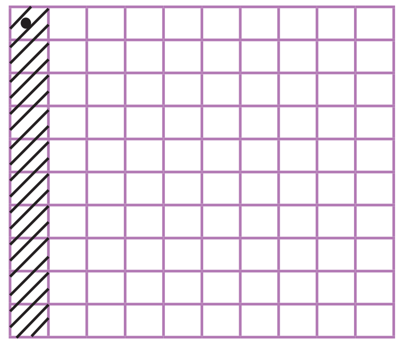
At first, let's see the picture below:

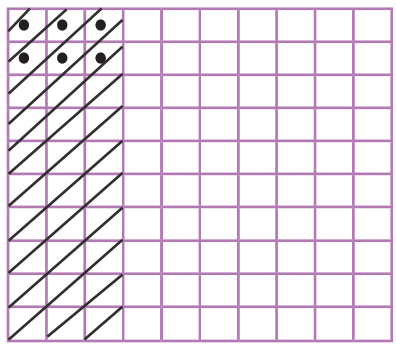


The rectangle in the picture is divided into 10 equal parts. The marked part indicates 1/10 part of the whole rectangle. We know, $\frac{1}{10} = 0.1$

Now if we divide $\frac{1}{10}$ of the whole rectangle into 10 more equal parts like the picture below, then the dotted square will be $\frac{1}{10}$ of $\frac{1}{10}$ part of the whole rectangle. That is,

$$0.1 \times 0.1 = \frac{1}{10} \times \frac{1}{10} = \frac{1 \times 1}{10 \times 10} = \frac{1}{100} = 0.01$$





Now let's find the product of 0.2×0.3 through the grid.

$$0.2 \times 0.3 = \frac{2}{10} \times \frac{3}{10} = \frac{2 \times 3}{10 \times 10} = (2 \times 3) \times \frac{1}{100}$$

$$= 6 \times \frac{1}{100} = \frac{6}{100} = 0.06$$

Then, From the method of multiplying by grid we found a simple way to find the product of $0.2 \times 0.3 = 0.06$.

Numbers need to be multiplied like normal multiplication without thinking about decimal points. For example: $2 \times 3 = 6$
After calculating the digits in regard to decimal points' place in both multiplier and multiplicand, the number of digits after decimal point have to be calculated, and from the right side of the product, the decimal point should be placed to the left of total calculated digits after decimal point. That means, $0.2 \times 0.3 = 0.06$

And this is the traditional method of multiplying a decimal fraction by another decimal fraction.

Individual task : Solve through drawing grid.

a) 0.2×0.4 b) 0.5×0.8 c) 0.6×0.4 d) 0.8×0.5 e) 0.7×0.3

Division of decimals

Let's figure out - how to divide a decimal fraction with another decimal fraction? Is it like the division of integers, or in some other way? Let us think of a solution to the following mathematical problem.

$1.2 \div 0.3 = ?$

You already know $1.2 = \frac{12}{10}$ and $0.3 = \frac{3}{10}$

Now, $1.2 \div 0.3 = \frac{12}{10} \div \frac{3}{10} = \frac{12}{10} \times \frac{10}{3} = \frac{12}{3} = 4$

We can get the idea of dividing the decimal to the decimal in another way.

Multiplying or dividing the divisor and the dividend by the same number does not change the quotient.

Let us try to divide the decimal by the decimal using this rule.

$1.2 \div 0.3 = (1.2 \times 10) \div (0.3 \times 10) = 12 \div 3 = 4$

From the above discussion we found an easy way to find the quotient of $1.2 \div 0.3 = 4$.

- Multiplying the divisor and dividend by the same number we need to try to get both to numbers to integers.
- In this case it is necessary to see if the number of digits is equal after the decimal point of the divisor and the dividend.
- Then the divisor and dividend have to be multiplied by 10, 100, 1000 etc. accordingly.
- Then we need to find out the quotient as usual.

Pair work : Each of the pair should create five problems like this. After solving the problems, exchange the exercise books. Find out each other's mistakes. Correct the mistakes through discussion. If needed, take help from the teacher.

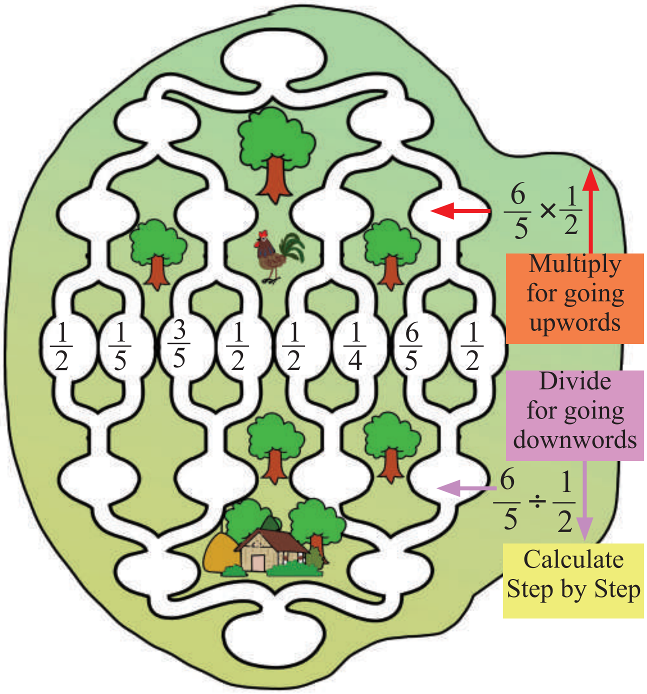
Individual task : Solve through grid.

a) $4.5 \div 1.5$ b) $9.12 \div 0.06$ c) $10.4 \div 2.6$

d) $9.5 \div 0.38$ e) $0.09 \div 0.03$

Exercise

1. Use the fractions in the middle of the image. When you move upwards, multiply each pair to fill in the blanks and when you go downwards, fill in the blanks by dividing the left fraction by the right fraction of each pair. Like this, find the last fraction of both the top and bottom positions.

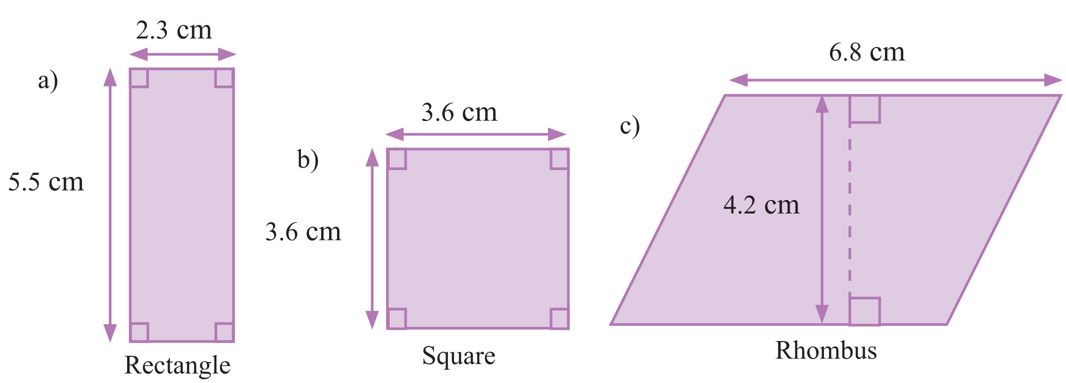


2. Ria is interested to put up a fence on three sides of her garden. The lengths of these three sides are 15 meter, 13.5 metre and 12.3 metre. It costs Ria 75.75 taka per metre of fencing to put up the fence.

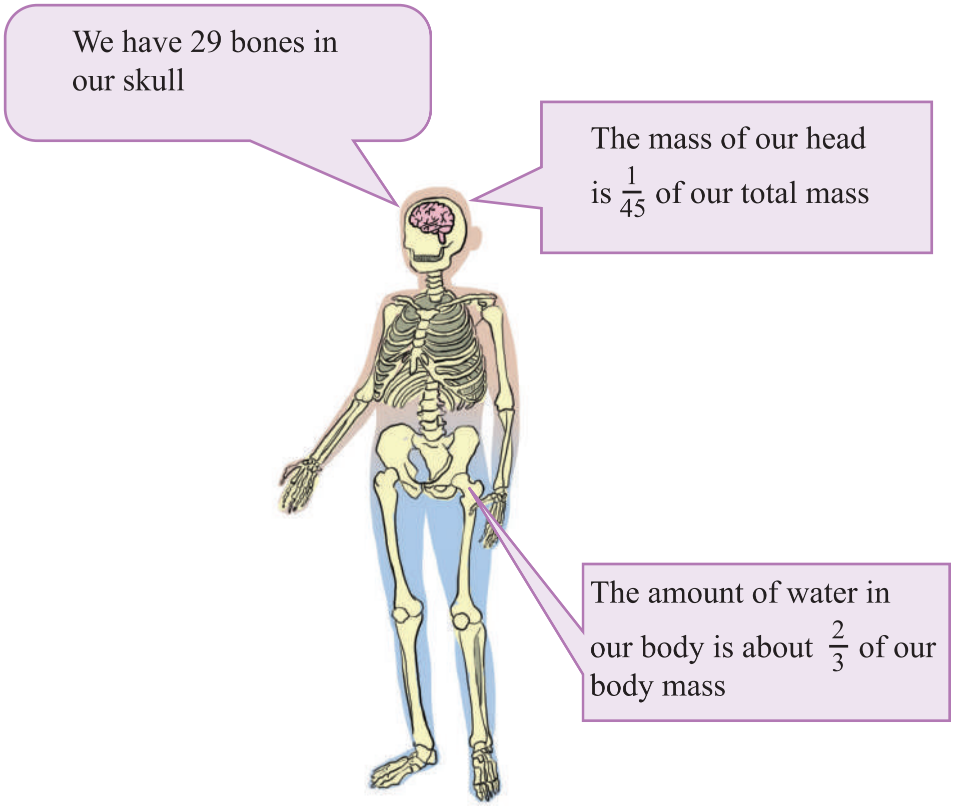
   a. How many metres of fencing will Ria need to put up?

   b. How much would the fence cost Ria?

3. Determine the range and area of the following diagrams:



4.



Observe the above diagram and think about our body.

a. What is the mass of your head?

b. If the number of bones in your head are $\frac{1}{7}$ of total bones in your body, how many bones do you have?

c. How many kgs of water does our body need to remain healthy?

5. Ratul planted some young flower saplings along the length and width of his rectangular garden, planting four and three saplings in each row, respectively. The distance between two consecutive saplings is $\frac{2}{3}$ metre. Draw a picture and think about it.

a. Find the area of Ratul's garden?

b. How many flower saplings did Ratul plant?many bones do you have?

6. Ria's family has 8 members. Ria made 0.56 litres of tea to serve equally amongst all. But Ria does not drink tea. How many litres of tea will be in each cup?

7. Ratul bought 1.5 kg of lentils at Tk 105 per kg, 5 kg onions at Tk 45.50 per kg from the market. How much will he pay the shopkeeper?

8. Shumon can travel 8 km distance per hour by a cycle.

   a) How many km of distance can Shumon go in 6 hours?

   b) How many hours will Shumon take to travel a distance of 30 km?

9. Auhona used the following ingredients of salad to make salad for her younger brother and herself:

Ingredients	Amount
Tomato	$\frac{1}{2}$ kg
Cucumber	$\frac{1}{4}$ kg
Onion	$\frac{1}{20}$ kg
Green chili	$\frac{1}{100}$ kg
Coriander leaves	$\frac{1}{125}$ kg
Salt	$\frac{1}{500}$ kg

   a) Determine the total weight of Auhona's salad in kg.

   b) If you have to make the salad for 5 members of the family, including the parents, present the necessary ingredients of the salad in a Table form and find out the total weight in kg, of the salad made.

World of Unknown Expressions

Abu-Abdullah Muhammad Musa al-Khwarizmi (780-850 AD)
Reference : Wikipedia

We have learnt about numbers and their characteristics. We have also learnt to solve various mathematical problems using numbers. Besides, we have learnt about the shapes of two-dimensional and three-dimensional objects. We can measure the perimeter, area and volume of some objects. Now we shall learn about algebra, one of the important branches of mathematics. **Algebra** is an old treatise and one of the basic branches of mathematics. The English word algebra has come from Arabic word "Al-Jabur". The famous Persian Mathematician **Abu-Abdullah Muhammad Musa al-Khwarizmi** (780-850) used the word in 820(AD) in one of his famous books.

Al-Khwarizmi was a mathematician, geographer and astrologer at the same time. But basically, he is mostly known for algebra. For this reason, he is known as the father of algebra.

Use of Algebra

Probably you are thinking, why should we learn algebra, right? Are there any uses of algebra in our real life? The answer is 'yes'. Algebra is used everywhere in our daily life. Starting from our cooking in the house, it is used in various fields, such as business, science, engineering etc.

Many of you must be surprised, where is the use of algebra in the field of cooking? Your mother regularly cooks for you; does she do the same even when you have guests in the house? Or does she change the ratios of the familiar ingredients? Have you ever thought, how does your mother maintain the consistency of the recipe? Even if you think it is hilarious, your mother has used algebra here.

If you take a loan from or invest money in any financial organization, you must count the interest or profit. Algebraic formulae are used to compute the long-term profit.

Simply stated, as algebra bridges between all branches of mathematics, similarly, it maintains an important role in almost every stage of our daily life.

What'd you say if we start with a game?

Rules of the game:

- Write a number in your exercise book according to your choice. The number can be a whole number or a fraction or in any other form.
- Now multiply the number of your choice written in your exercise book by 3.
- Add 30 to the result obtained after multiplication.
- Divide the result by 3.
- Subtract the number of your choice from the quotient obtained.

If your friend knows the game, then he/she can tell what the subtracted result is. Although your friend does not know the number of your choice, he/she can tell you – the subtracted result will be 10.

The game is not very complicated. If you think a little bit, you will understand how your friend could tell the subtracted answer without looking at the number you wrote.

$$\frac{\boxed{} \times 3 + 30}{3} - \boxed{} = 10$$

Now have a look, if you arrange the rules of the above game, they become as follows—

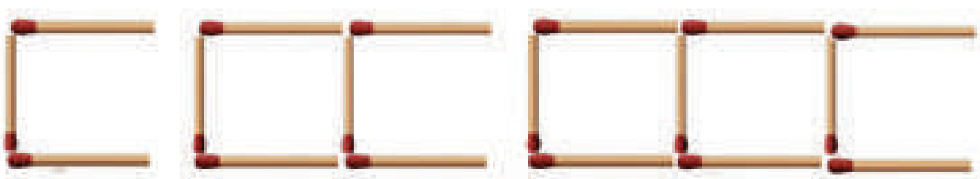
Considering any number in the blank space or multiplying, adding, and dividing by any number, you can play the game. Do you want to try?

Pair work : Play the game several times with your classmate. You may also play with your family members or neighbours.

Algebraic symbol and variable

The main characteristics of algebra are the use of letter symbols. Using the letter symbols, we can consider any number instead of a fixed number. Surely you remember we used 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 as number symbols or digits in arithmetic. The number symbols or the digits in algebra are 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. Besides, letter symbols are also used in algebra together with the number symbols. And known or unknown number or quantity is expressed by the English lower-case letters.

In the diagram below, Samir and Anannya made a pattern of the English letter C using matchsticks. Samir used 3 sticks (fig 1) to make the first C. Annanya added 3 more sticks (fig2) to the C Samira made. In this way, both (fig 3) are making more C's.



1 2 3

Their friend Amiya joins at this stage. Looking at the patterns, she asks Samir and Anannya how many sticks will be required to make fig 6? Then Samir and Anannya prepare the following table:

Figure no	1	2	3	4	5	6	7	-	-	-
Necessary matchsticks	3	6	9	12	15	18	21	-	-	-

fig- 1

Amiya got her answer by looking at the Table. She said 18 sticks will be needed for the 6^{th} pattern.

Samir and Anannya could realize during making the table that the number of sticks needed was 3 times the picture number. That is number of sticks needed = 3 × picture number.

If the number of pictures is expressed by n, then for the first C, n = 1, for the second C, n = 2, for the third C, n = 3, ...

So, the number of the picture n = 1, 2, 3, ... etc, are natural numbers. According to the table, number of sticks = 3 × n = 3n and this is a rule.

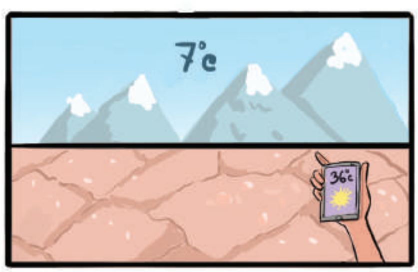
Anannya said, I can tell you how many matchsticks will be needed to make the 100th figure using this rule. In this case, I shall not need to prepare a table. Amiya and Samir both agreed with Anannya.

We can see from the above that, if n changes, number of sticks needed also changes. That is n is not fixed value. It can attain any value. n is an example of a variable. You may have a query; can we not use any other letter than n as a variable?

Of course, you can. Instead of the symbol n, x, y, z, ... etc symbols may also be used. We also find use of variables in real life.

Let us observe the following picture and try to find the answers to the following questions.

Speed of car



Temperatures of different places

Growth of children with time

- Does the speed of car remain same with change of time?
- Does the daily temperature of different places of the world change?
- Does the growth of a child change with the change of time?
- Does the age of human being increase or decrease year after year?

None of the events of the above picture is fixed. That is all the numbers used are changeable. Hence, we can name these numbers as **variables.** Value of the variable changes with place and time.

Work in Pairs: Make patterns of the English letter F using matchsticks like Samir and Anannya. Then show the pattern using a table. By observation of the Table, find a formula or a rule to express the relationship between the picture and the number of sticks needed. Using the formula, find the number of sticks for the 120th picture.

Let's learn more about variables

Let us try to understand variable through an example. What is the number of daily presences in your class? Surely the number is a wanderer. That is, it is not same every day. The number may be 0, if all in the class have made a pact not to come, again all of them in the class may be present on a exam day. Although the total number of students in your class is fixed, daily presence will change (on different days). Hence, we can name this quantity of **presence** as a **variable** and we say humorously, "since it changes instead of staying fixed, it is called variable, since it varies".

Variable

1. Variable is such a symbol whose value changes.
2. The value of a variable is not fixed.
3. Variable can have different values.

Constant

If we know light, we have to know darkness too. Similarly, if we know variable, we need to know about constant too. Constant is a measurable quantity too, like variable, the value of which is not changeable. The numbers we work with: 1, 2, 3, 4, ………., 100,……,500,…….., 1000000, ………., each one of them is a constant, since there is no change in their values.

One morning, if a sad friend of yours tells you "I saw one stirling", then you will imagine exactly one stirling, not 5 or 10. These numbers are without a unit, it is not difficult to find constants with units. For example, if we say that the speed of sound is 332 m/s through air at 0^0 C, then you will think sound travels in this fixed speed.

Symbols of Operations:

Earlier, we have learnt about the operations of addition, subtraction, multiplication, division, greater and smaller in Arithmetic. Symbols used for these operations are called the symbols of operations.

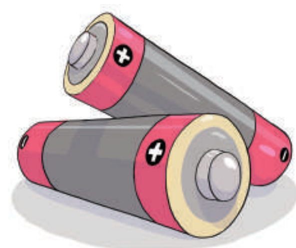
Observe the following Table:

Operation symbol in arithmetic	+	-	×	÷	>	<
	Addition	Subtraction	Multiplication	Division	Bigger	Smaller
Operation symbol in algebra	+	−	×, ·	÷	>	<
	Plus	Minus	Into or dot	Division	Grater than	Less than

fig- 2

Application of Symbols of Operation

Can you name an item where + and – symbols are used? Surely you can name the two objects in this picture. Can you think where this is used? Can you name some other objects where the operation symbols may be used?



And now let us use the operation symbols to make different relationships between the variables x and y, in the following table:

Serial number	relation between x and y (in words)	relation between x and y (using operation symbols)
(i)	x plus y	$x + y$
(ii)	x minus y	$x - y$
(iii)	x times y	$x \times y$ or $x \cdot y$ or xy
(iv)	x division y	$x \div y$ or $\frac{x}{y}$
(v)	x times 5	$x \times 5$ or $x \cdot 5$ or $5x$ but not $x5$ for multiplication first number is written, then the letter e.g. 3x, 10y, 9z etc.
(vi)	x is bigger than y	$x > y$
(vii)	x is lesser than y	$x < y$

fig- 3

Algebraic Expression, Term and Coefficient

In the story of arithmetic or numbers, you have created numerous mathematical relationships combining two or more numbers. For example:

$3+(4\times5)-6$, $100-25+8\div7$ etc.

These relationships have been made using the digits or numbers 3, 4, 5, 100, 25, 8,7 etc.

Observe that the digits or numbers have used the operations +, $-$, $\times$, $\div$, etc.

Similarly, using operation symbols, numerical signs or variables, constants etc, in Algebra, a type of quantity is made, known as **Algebraic Expression**. Remember one thing, "There must be one or more variables in an algebraic expression":

Example: $2x+5$, $3x+2y$, $5x-7y+z$, $8x\div12y-16y\times6z$, etc.

Pair work : Construct at least 10 Algebraic Expressions using several variables and write them in your exercise books. Then exchange copies of each other and identify mistakes, if any.

Pair work : Construct at least 10 Algebraic Expressions using several variables and write them in your exercise books. Then exchange copies of each other and identify mistakes, if any.

Serial no	Problem	Algebraic/Arithmetic expression	Logical Explanation
(i)	You know your age. Age of your mother is 2 years more than four times your age.		
(ii)	The cost of one Kg of rice is Tk 30 less than the cost of one Kg of lentil.		
(iii)	The present age of Shila's father is four times of Shila's age. The age of Shila's grandfather is fifteen years more than the sum of Shila's and her father's age. How old is Shila's grandfather?		

Serial no	Problem	Algebraic/Arithmetic expression	Logical Explanation
(iv)	 Total number of apples if each box has 50 apples		
(v)	The speed of a bus per hour in a highway is 12 km more than the speed of a truck		
(vi)	Four times of a number minus three times of another number		
(vii)	Nafisa made oral saline using proportionate amount of water, molasses and salt		
$(viii)$	Total cost of ten exercise books, five pens and three pencils		
(ix)	Amena has some chocolates. Liana has 5 more than Amena. Liton has 7 chocolates. What is the total number of chocolates among the three?		

fig- 1

Complete the following table:

Serial no	General description	Variable	Expression through algebraic quantity
(i)	Oishi has 5 more chocolate than Mita	Suppose Mita has x chocolate	Number of chocolates Oishi has is (x+5)
(ii)	Binoy is 11 years younger than Manik	Suppose Manik is x years old	
(iii)	Rifa has tk 15 more than the double of Kajol has	Suppose Kajol has tk y	
(iv)	How old will Bikash be after 4 years?	Suppose present age of Bikash is x	
(v)	How old was Lamia 7 years ago?	Suppose present age of Lamia is y years	
(vi)	Shihab obtained 3 more marks than half of Matin's mark in mathematics	Suppose marks obtained by Matin is x	
(vii)	What is the perimeter of a rectangular garden if the length is double the width?	Suppose width of the garden is y	
(viii)	If 4 students sit on a bench, 3 benches remain empty. How many benches are there in your classroom?	Suppose there are x students in your class	
(ix)	If 3 persons sit on a bench, 6 students remain standing. In this case how many benches are there in your class?		
(x)	Mr Rahim gave Tk 500 to his friend from his savings		
(xi)	Mr David had some money in the bank. He deposited Tk 1000 more in the bank.		

fig- 5

Term

Each part of an algebraic expression connected only a by plus sign is called a **Term.**

Example: $2x$, $5x + 2yz$, $3x - 2yz + 7a \div 9$ etc.

Here there is one term $2x$ in the first expression, two terms $5x$ and $2yz$ in the second expression and three terms $3x$, $-2yz$ and $7a \div 9$ in the third expression.

You must be wondering, $-2yz$ in the expression $3x - 2yz + 7a \div 9$ is not connected by a plus sign. How is this a term then?

Let us rearrange the expression as follows again:

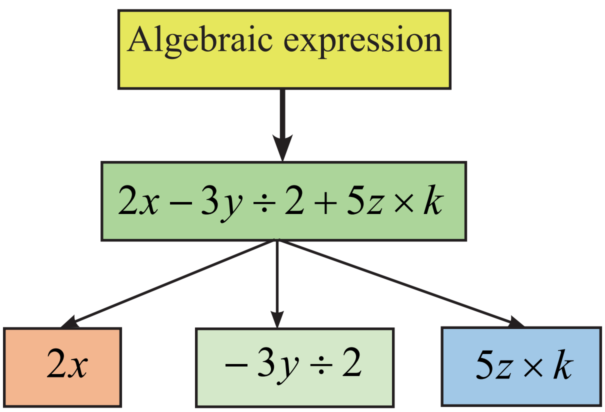
$3x + (-2yz) + (7a \div 9)$

So now we can say, **"The terms in algebraic expression are connected by plus sign"**.

Individual tasrk : How many terms are here in the following expressions? Find the terms:

(i) $2x + 4y - 5z$, (ii) $7a - 5bc + 8d \div m$, (iii) $14x - 5y$

If there is more than one term in some algebraic expression, then we can separate them like a tree in the following diagram:



Work in Pair : Write down at least 3 algebraic expressions involving three terms, at least 2 algebraic expressions involving four terms and separate the terms using tree diagram.

Individual task : Complete the following Table:

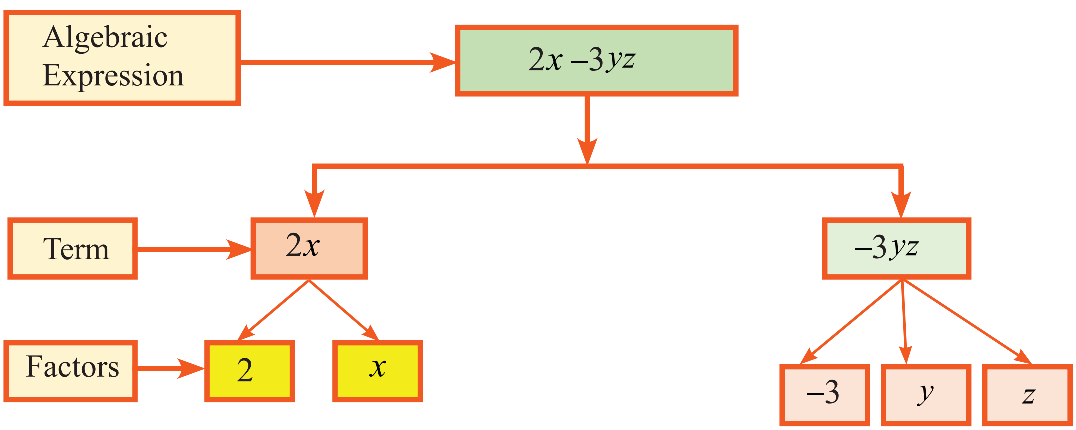
Serial	General description	write using $+, -, \times, \div$	Number of terms	The terms are
(i)	Subtract three times y from five times x			
(ii)	Add four times c to product of a and b			
(iii)	Subtract 3 from product of x and 12			
(iv)	Divide 3 by x, 7 by y and 9 by z and add the quotients			
(v)	Divide the sum of p and q by r			

fig- 6

Factors of a term

We have already learnt that there are the two terms 5x and – 2yz in the expression 5x – 2yz.

Here the factors of 5x are 5 and x and the term – 2yz is the product of – 2, y, z. Very easily, we can express the terms of any algebraic expression by tree as in the following diagram.



Coefficient

We have learnt how we can write the terms as a product of two or more factors. We have also understood which factors of the term are numbers and which are algebraic expression or symbol. If a number is included as a multiplier with the variable of some term, then that multiplier will be called the numerical coefficient or just **coefficient**.

Example: The numerical coefficients of $4x$, $6xy$, $-15xyz$ are 4, 6, -15 respectively.

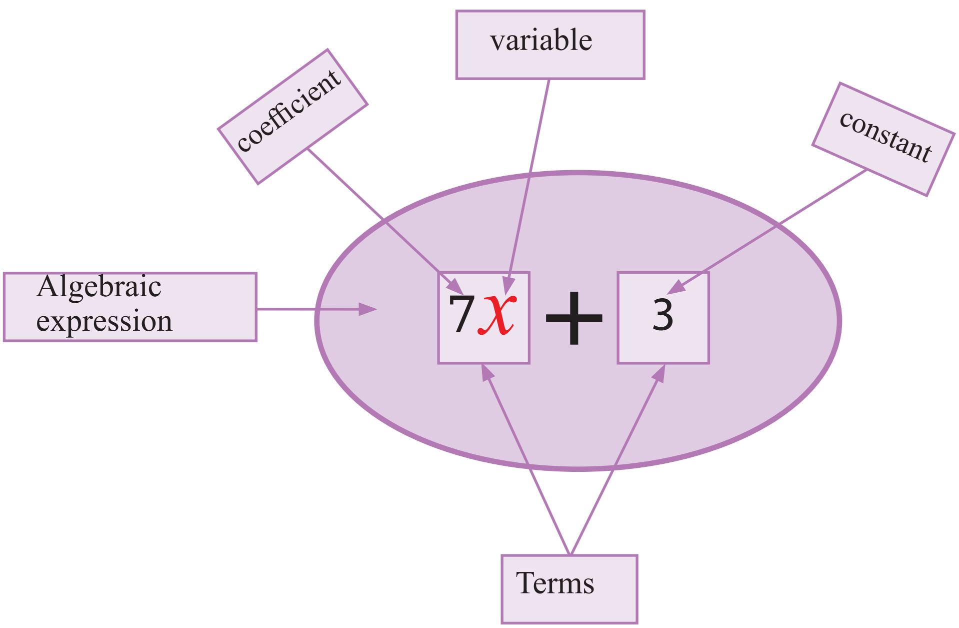
Individual task : Write down one algebraic expression of three terms and one of four terms. Then show the factors of each term using tree.

If there is no numeric multiplier included in any term of an algebraic expression, then the coefficient of that term is considered to be 1. As we write $1x$ only as x, $-1xy$ only as $-xy$ etc. hence the coefficients of x and $-xy$ are 1 and -1 respectively.

When a letter symbol is included as a multiplier with a variable, then that multiplier is called the letter coefficient of that expression or term.

Suppose $10abc$ is a one term expression. Here 10 is the numerical coefficient of abc, a of $10bc$, b of $10ac$ and c of $10ab$ are the letter coefficients.

Now let us have a quick look at what we get when we split up an algebraic expression:



Individual Task : Complete the following Table:

Serial	Algebraic Expression	Term with x	Coefficient	Term with y	Coefficient
(i)	$3x - 4yz$	$3x$	3	$-4yz$	$-4z$
(ii)	$5 - x + 7aby$	$-x$	-1	$7aby$	$7ab$
(iii)	$px - \frac{2}{3}y$				
(iv)	$abx + 23$				
(v)	$9 - 11bz$				
(vi)	$4x + 12y - 14z$				
(vii)	pxy				

fig- 7

LIKE AND UNLIKE TERMS

Samira and Anannya went to a shop. Samira bought five pens and three exercise books and Anannya bought four pens and two pencils from the shop.

Surely, you can tell the similarities or differences of the items both bought. The similar item (pen) both bought is the 'like' item. Both bought two more different items (exercise book and pencil). Then those two different things are 'unlike' items. So, you get some idea about 'like' and 'unlike' items. Now let's try to find like and unlike terms in algebraic expressions.

Thoroughly examine the following algebraic expressions:

$(i)\ 2x + 3x \quad (ii)\ 5aby - 7yba \quad (iii) - xyz + 11yxz$

In number (i), factors of 2x are 2, x and factors of 3x are 3, x. It is apparent that the algebraic factors of both are same. That is, the only difference in the two terms is their numerical coefficients. This type of terms is called like terms.
Similarly, can you think about, if the terms of the expressions in (ii) and (iii) will be like terms or unlike?
Again, observing the expressions
$(iv)\ 3xy - 2y$ $(v)\ 13p + 13q$ $(vi)\ 2ab + 5a - 19c$, we can see that the algebraic factors of terms 3xy and $-2y$ in number (iv) are different. Hence, we say that these terms are unlike. If the algebraic factors of several terms are different, then the terms are unlike even if their numerical coefficients are same.
For example: In $(v)\ 13p + 13q$, the terms 13p and 13q are unlike.

Pair work : Write down at least 5 like and 5 unlike terms individually. Then interchange your notebook with your pair and discuss about the errors. Then correct them.

Individual task : Explain, with reasons, if the two terms given in the following table are like or unlike:

Serial	Pair of terms	Factors	like/ unlike	Logical Explanation
(i)	$3x, 4x$	$\left.\begin{matrix}3, x\\4, x\end{matrix}\right\}$	Like	algebraic factors of both are same
(ii)	$5ax, 7aby$	$\left.\begin{matrix}5, a, x\\7, a, b, y\end{matrix}\right\}$	unlike	algebraic factors of both are different
(iii)	$11xy, -\frac{2}{3}yx$			
(iv)	$abx, 23axz$			
(v)	$-17bz, 25az$			
(vi)	$4x, 12y$			
(vii)	pab, qba			
$(viii)$	$\frac{7}{9}mn, -13nm$			

fig- 8

Addition of Algebraic Expressions

We have known that Samira bought five pens and three exercise books and Anannya bought four pens and two pencils from the shop. If you are asked, how many items in total did they buy? All of you will probably answer nine pens, three exercise books and two pencils. Think about it once – you have added only the pens both bought, and said nine pens, the remaining two items you quoted separately. That is, you can only add the same or like items, and the unlike items are added separately.

Now let us learn how to add two or more algebraic expressions. And for this reason, you need to be able to add numbers with signs.

We have of course learnt how to add numbers with signs in the previous chapter.

Example: $5+3=8$, $5+(-3)=2$, $-5+3=-2$, $-5+(-3)=-8$ etc.

Again, we discussed in detail, about coefficients of algebraic terms, like terms and unlike terms.

Now if we want to add two or more algebraic terms, then first add the signed numeric coefficients of the like terms only. Then write the letter symbols in the right of the newly calculated coefficients.
Question is, what will happen to the unlike term or terms?
You have to include the unlike term or terms together with their signs, with the sum. Then you will get the sum of two or more algebraic terms.
Let's try to understand the matter with the help of examples:

- Suppose $7x$ and $9x$ are two terms. Obviously, they are like terms. Hence sum of the two terms $= 7x + 9x$

  $= (7+9)x$

  $= 16x$

- Let us give another example. $2xy, -3xy, 6xy$ and $11z$ are four terms. Are they all like terms? Think about it.

  Sum of the terms $= 2xy - 3xy + 6xy + 11z$

  $= (2-3+6)xy + 11z$

  $= (8-3)xy + 11z$

  $= 5xy + 11z$

Now we shall discuss how to find the sum of two or more algebraic expressions.

Suppose $20ab + 15b + 12a$ and $4ab - 11b - 14a$ are two Algebraic expressions. Have to find the sum of the two expressions.

First Method :

Sum $= (20ab + 15b + 12a) + (4ab - 11b - 14a)$

$$= (20ab + 4ab) + (15b - 11b) + (12a - 14a)$$

$$= (20 + 4)ab + (15 - 11)b + (12 - 14)a$$
$$= 24ab + 4b + (-2)a$$

$$= 24ab + 4b - 2a$$

Second Method :

Write the like terms with their signs under each,

$$\begin{array}{r} 20ab + 15b + 12a \\ \underline{+ \quad 4ab - 11b - 14a} \\ 24ab + 4b - 2a \end{array}$$

Sum obtained : $24ab + 4b - 2a$

Work in pairs : Each of you construct at least three algebraic expressions containing three to four like and unlike terms including plus-minus symbols. Then compute the sum of the expressions and exchange your exercise books. Mark each other's mistakes (if any) and do the corrections through discussions. Take help if necessary.

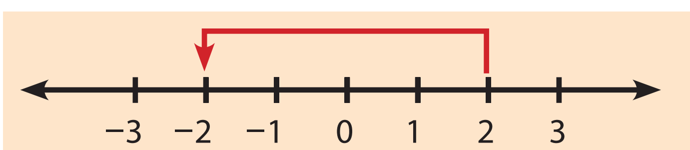
Subtraction of Algebraic Expressions

In previous chapters, we have learnt about the Additive Inverse. Let us refresh it a bit. If sum of two numbers is zero (0), then one will be called the **Additive Inverse** of the other.

Example: $3 + (-3) = 0,\ 7 + (-7) = 0$

Here the additive inverse of 3 is -3 । Similarly the additive inverse of 7 is -7

The additive inverse of 2 is –2



Can you tell what the additive inverse of 0 is?

x is a quantity and since $x + (-x) = 0$, hence the additive inverse of x is $-x$.

Similarly, the additive inverse of $a - b$ is $-a + b$

Since $a - b + (-a + b) = a - b - a + b = (a - a) + (b - b) = 0 + 0 = 0$

Now we shall discuss how to subtract an algebraic expression from another algebraic expression.

Subtracting an algebraic expression from another algebraic expression means, to add the additive inverse of the second expression to the first expression. This means, changing the signs of each term of the second expression and then adding to the first.

We shall explain this with the help of an example:

Suppose we have to subtract $3x - 4y - 6z$ *from* $5x + 4y - 5z$.

First Method :

The additive inverse of

$3x - 4y - 6z$ *is* $-3x + 4y + 6z$

Hence adding the additive inverse of the second to the first expression, by writing the similar terms under each, we get

$$\begin{array}{r} 5x+4y-5z \\ -3x+4y+6Z \\ \hline 2x+8y+z \end{array}$$

Second Method :

$$\begin{array}{r} 5x+4y-5z \\ 3x-4y-6x \\ (-) \quad (+) \quad (+) \\ \hline 2x+8y+z \end{array}$$

Here also the like terms have been written underneath their counterparts changing the signs of each term of second expression and added.

Work in Pairs : Each one of you construct two Algebraic expressions containing three or four terms with two like terms, having plus-minus signs. Then subtract the second expression from the first expression and exchange the exercise books. Identify the mistakes (if any) of each other and correct them through discussions. You can get help from your teacher if necessary.

Exercise

1. What is meant by the following algebraic expressions?
   $(i)\ 7x$ $(ii)\ 3x+5$ $(iii)\ 4x-11y$ $(iv)\ \frac{1}{2}(2x+3y)$ $(v)\ \frac{x}{2}+\frac{y}{3}-\frac{z}{5}$
   $(vi)\ 12x-13y+15z$ $(vii)\ \frac{2}{3}(x+y+z)$
2. Express the following relations in algebraic expressions, using the processing symbols:
   I. Add four times y to five times x
   II. Subtract 3 times a number from double of another number
   III. Shawpna bought four oranges at the rate of x taka per dozen, one dozen banana at the rate of y taka per four, from a shop. How much did Shawpna spend?
   IV. Multiply a by b, then divide the product obtained by 7 times c.
   V. If there are x number of bubble gums in a packet, then how many bubble gums are there in the picture?

   VI. Robin bought five chocolates for his sister and three for each of his friends. How many chocolates did he buy in total?
3. The cost of an exercise book is x taka, cost of a pencil is y taka and cost of an eraser is z taka.
   a) How much did Mita spend, buying one dozen exercise books and half a dozen pencils?
   b) Shajib bought eight pencils and two erasers. How much did he spend?
   c) Pryanka bought three exercise books, four pencils and one eraser and gave the shopkeeper a 100 taka note. How much did the shopkeeper return Pryanka?
4. Add the following:
   $(i)\ 2a+3b, -a-2b$
   $(ii)\ 4x-5y, -2x+y, 6x+7y$
   $(iii)\ 7x+5y+2z, 3x-6y+7z, -9x+4y+z$
   $(iv)\ 5ax+3by-14cz, -11by-7ax-9cz, 3ax+6by-8cz$
   $(v)\ 12x+15y-10z,\ 15z-24x-9y, -6y+12x-5z$

5. Subtract the second expression from the first expression:

   (i) $12a + 23b, 7a - 2b$

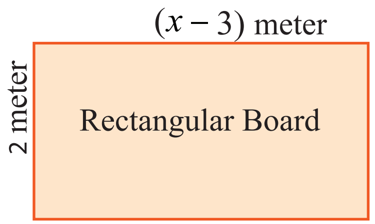
   (ii) $4x - 5y,\ 6x + 7y$

   (iii) $10x + 5y + 20z,\ -9x + 4y + 25z$

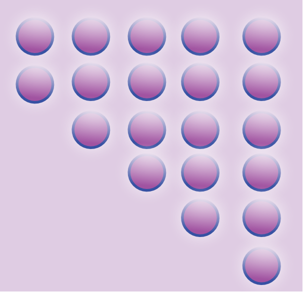
   (iv) $5px + 8qy - 14rz, -11qy - 7px + 9crz$

   (v) $20x - 5y + 30z,\ 15z + 4x - 9y$

6. a) Find the perimeter of the board
   b) Find the area of the board.



7. The following picture is a pattern made by marbles. How many marbles will be required to make the 100^{th} column?



8. Suppose you want to make a soup of your choice at home. Make a list of the ingredients you will need for that. If many people want to have that soup, then express the ingredients of the soup and the number of people using an algebraic expression.

9. If $x = 5a + 7b + 9c,\ y = b - 3a - 4c,\ z = c - 2b + a$ then show that, $x + y + z = 3(a + 2b + 2c)$

Linear Equation

Do you know the name of the object in the diagram on the side?
This is called balance. You will find this used for weighing different commodities for selling in shops. At present, we usually say the weight of an object is 1 kg, 2 kg etc. But saying 'weight 1 kg' is really not correct. You will notice things are weighed in grams or kg (kilogram) units. So, you understand, it is not the weight, but mass is being weighed.

If you want to understand this matter well, study the topic 'measurement of different quantities' in 'Chapter 1- Science and Technology' of Class Six science textbook. Do you know how objects are weighed using a balance? The balance has two scales, one pan at the left, one at the right. The scale pan which has more weight has more mass. Hence that side becomes down. That is the scale pan which has things of less weight, goes up. Example:

A shopkeeper put a weight of 5 kg on the left scale pan and some potato on the right.
Are the weights of the two scale pans equal?
Will it be possible here to say what the exact weight of potato is?

Then we can say the weight of potato is not known or unknown.

Now the shopkeeper put a weight of 2 kg in the right scale pan with the potato. As a result, the weights of the things on both scale pans are equal.

Now if we assume that the unknown weight of potato is x, then the total weight on the right scale pan is $(x + 2)$ kg.

Then the equilibrium of the two scale pans can be expressed as an algebraic relation and that is: $x + 2 = 5$

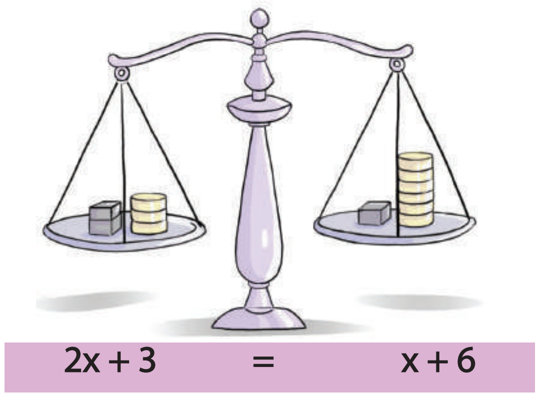
This is a mathematical statement and equality. Mathematical statement with the sign of equality is called an Equation. Here we say, the unknown quantity x is a variable. Usually, the small letters in the English alphabets are used for the unknown quantity or the variable.

Now think if you can find any similarities between the 'balance' and 'equation'. The balance has two scale pans. One is the left scale pan; the other is the right scale pan. The balance becomes equilibrium if both the scale pans have equal weights. If you reduce the weight of one scale pan, the other will come down. That means, weight on that side becomes more. In that case the balance is not in equilibrium position.

Can you imagine how you can bring the balance back to equilibrium again? Probably you are thinking right – there are two ways you can bring the balance back to equilibrium.

1. Reducing the weight from the scale pan, which went down or
2. Increasing the weight of the scale pan which went up.

On the other hand, an equation also has two sides. One is the left side, and the other is right side. There is an (=) sign in between them. The quantity on the left side of the equal sign is called Left Hand Side, and the quantity on the right side is called Right Hand Side. The left hand side and the right hand side of the equation must be equal for a fixed value of the variable.



We can cite some examples: $x + 4 = 13$, $x + 6 = 9$, $2y - 1 = 5$, $3 - z = 10$ etc are equations. Here *x, y and z* have been used as variables and for fixed values of the variables, left hand side and right hand side of the equations are equal.

Individual Task : Each one of you write five equations using the variables *x, y* and z.

Let's learn more about Equations

Many among you have queries about equations. Then let us try to understand the topic through a story. Suppose Shopnil is 2 years younger to Mita. If Mita is x years old, then the age of Shopnil will be $(x - 2)$, is it not? Now suppose, Shopnil is 12 years old. Then, surely there is a relationship between $(x - 2)$ and 12. The relationship is $x - 2 = 12$. This is an equation in variable x.

Now let us fill up the following Table for the values of (x – 2) for different values of x:

x	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$x-2$	0	–	–	–	–	–	–	–	–	–	10	11	12	13	–	–

Fill up the empty spaces of the table. Note the table, you will find that the left hand side and the right hand side are equal for the relation $x - 2 = 2$ only for the value x = 14. For any other value of x, e.g. $x = 12$, $x = 16$ etc, the left hand side and right hand side are not equal for the relation $x - 2 = 12$. At last we can say that we shall call it an equation only if the left hand side and right hand side are equal for a fixed value of the variable.

Surely you know that an algebraic expression with variables can also be expressed using the symbols greater than or smaller/less than .

Example: $x-1>8, 2y+7<13,\ z>15$ etc. But these types of algebraic expressions with variables cannot be termed as equations. Think a little bit, and say, why they cannot be called equations? Does the symbol (=) exist? Are they valid for a fixed value of the variable? Of course not, right? The expressions with the symbols 'greater than' or 'smaller than' are valid when variables have numerous values.

Observe the relation below:

$15+7=21$' Is this an equation? The symbol (=) is there in the relation! Think and answer.

Fill up the following table:

Serial	Algebraic relation	Unknown quantity or variable	Put (if equation if not put (x)	Comments with logical explanation
(i)	$x+20=60$			
(ii)	$2z>14$			
(iii)	$5y=100$			
(iv)	$\frac{x}{3}<1$			
(v)	$7-z=0$			
(vi)	$\frac{x}{0}=2$			
(vii)	$9-3=6$			

Individual Work : Make a grid like the above on your khatas. Then write down at least five algebraic relations and fill up the Table and submit it.

Linear Equation

A simple equation with unknown quantity or variable is known as a **Linear Equation.**

Example : $2x - 5 = 0, y + 3 = 10, 2x - 1 = x + 4$ etc. Because each of them is of one variable and each is linear.

Individual task : Write down at least five linear equations of one variable. Give logical explanations why the equation you wrote is a linear equation.

Expression of real life problems using linear equations in single variable

Use linear equations in single variable to express the real life problems in the following table and complete it. Here you may use the unknown quantity or the variable of your own choice.

Serial	Real life problems	Unknown quantity or variable	Equation
1.	Raju is 12 years old; Mita is three years younger to Raju.	Mita's age is x years	$x + 3 = 12$
2.	If 7 is added to the double of a number the number the sum will be 21	The number is y	
3.	4 chocolates remain with you after giving away 5 of your chocolates to your younger sister		
4.	The length of your rectangular classroom is 2 m more than the width and the perimeter is 60 m		
5.	Sadia has some money and Apu has tk 20. Both together have tk 45.		
6.	You had 15 plums, from which your friends ate some and 7 plums remain.	Number of eaten plums is x	

Group Task : The group leader will make a table similar to the table above in his/her exercise book. Then all the members of the group will discuss amongst them and write at least five real life problems and then will fill up the table.

Solution of Linear Equation

The process of finding the unknown quantity or the variable in an equation is called the **solution** of the equation. The value of the solution is the **root** of the equation. If the value of the root is substituted on both sides of the equation, then the left hand side and right hand side are equal.

Things to know for finding solutions of equations

1. If a quantity is added to each equal quantity of an equation, then the sums will be equal to each other.
2. If a quantity is subtracted from each equal quantity, then subtracted results will be equal to each other
3. If each equal quantity of an equation is multiplied by a quantity, then the products will be equal.
4. If each equal quantity is divided by a non-zero quantity, then the quotients will be equal.

Testing the solution by trial-and-error process we can reach the solution of linear equations.

Individual Task : Write an equation for each of the above four information given and solve the equations by simplifying.

Serial	Equation	Value of variable	Check accuracy	Put (✓) if solution correct put (x) otherwise
1.	$x + 5 = 9$	$x = 14$		
		$x = 4$		

Serial	Equation	Value of variable	Check accuracy	Put (✓) if solution correct put (x) otherwise
2.	$y - 6 = 11$	$y = 17$		
		$y = 5$		
3.	$2x + 1 = 25$	$x = 12$		
		$x = 13$		
4.	$\frac{y}{3} = 12$	$y = 4$		
		$y = 36$		
5.	$4 - x = 10$	$x = 14$		
		$x = -6$		
6.	$3z - 8 = z + 2$	$z = 5$		
		$z = 4$		

Exercises

1. Prepare a table and determine which of the followings are equations and which are not. Present with reasons.

$(a)\ 15 = x + 5$ $\quad (b)\ (y - 6) < 3$ $\quad (c)\ \frac{6}{3} = 2$

$(d)\ z - 4 = 0$ $\quad (e)\ (4 \times 3) - 12 = 0$ $\quad (f)\ 2x + 3 = x - 15$

$(g)\ y + 25 > 30$ $\quad (h)\ 8 - x = 11$ $\quad (i)\ 20 - (10 - 5) = 3 \times 5$

$(j)\ \frac{5}{0} = 5$ $\quad (k)\ 15y = 45$ $\quad (l)\ 7 = (11 \times 2) + x$

2. Express the following problems in the table below as equations:

Serial	Problems	Equations	Root of equation
(i)	7 added to the double of a number yields the sum 23		
(ii)	Sum of two consecutive natural numbers is 36 and the smaller one is y.		
(iii)	5 subtracted from four times of a number x, the subtracted result is 19 more than the double of the number.		
(iv)	The length of a rectangular pond is x m, width is 3 m less than the length and the perimeter of the pond is 26 m.		
(v)	Present age of son is y years. Age of father is 6 times the age of son. Sum of their ages is 35 years.		

3. Pick out the correct root from the values in the column beside each equation. Explain why the remaining values are not roots.

Serial	Equations	Values
(i)	$2x + 5 = 15$	10, 5, – 5
(ii)	$5 - y = 7$	12, 2, -2
(iii)	$5x - 2 = 3x + 8$	5, 1, -5
(iv)	$2y + 2 = 16$	18, 9, 7
(v)	$4z - 5 = 2z + 19$	12, 7, 4

4. Mina went to the market with a 100 taka note. She bought one dozen pens, each costing x *taka* from a shop. The shopkeeper returned her 40 taka. Mina bought y exercise books, each costing 12 taka from another shop and was left with 4 taka.

   a) Find the cost of each pen.

   b) How many exercise books did Mina buy?

5. Mr Karim invested some of his tk 56000 at 12% profit per annum and the remaining at 10% per annum. After one year he received total profit of tk 6400. How much money did he invest at 10% profit?

6. Shakib scored double the runs of Mushfiqur Rahim in a cricket match. Total runs of both were 2 short of double century. Who scored how many runs?

7. Fill up the empty squares

a)

$\square$	+	$\square$	= 10	
+		+		
$\square$	−	$\square$	= 12	
=		=		
17		10		

b)

$\square$	+	$\square$	= 15	
+		−		
$\square$	+	$\square$	= 15	
=		=		
12		2		

8. A water bottle weighs 150 gm. Mina put some water bottles in a bag which weighs 50 gm. The number of water bottles is denoted by x and the weights of the water bottles plus the weight of the bag is denoted by y.

   a) Write down the relation between x and y using an equation.

   b) Find the value of y when $x = 15$

   c) Find the value of x when $y = 1100$

9. The cost of x packets of biscuits and 1 bottle of drink together is y Taka. The cost of 1 packet of biscuit is tk 20 and cost of 1 bottle of drink is tk 15.

   a) Write down the relation between x and y using an equation.

   b) Find the value of y when $x = 25$.

   c) Find the value of x when $y = 255$.

10. The length of the playground of your school is 16 m more than the width.

   (a) If the width of the field is x m, find the perimeter of the field in terms of x.

   (b) If the perimeter of the field is 120 m, find the area of the field.

Story of Three Dimensional Objects

We have different shapes of two dimensional and three dimensional objects around us. Example: boxes of varied sizes, bricks, football, cricket ball, cupboard, papers, pages of exercise books, newspapers, matchbox, pipe, apple, orange, books etc. All the objects do not look alike, their characteristics are also different.

Can you talk about the characteristics of two dimensional and three dimensional objects?

What are the differences between the shapes of the two dimensional and three dimensional objects?

Observe the following pictures. Write down which one is two dimensional and which is three dimensional in the fixed boxes below in the table and draw a rough picture.

Name	Two dimensional	Three dimensional
Paper		

Individual task : Name at least 10 two dimensional and three dimensional geometric shaped objects found around you and bring their names with picture drawn, in the next class.

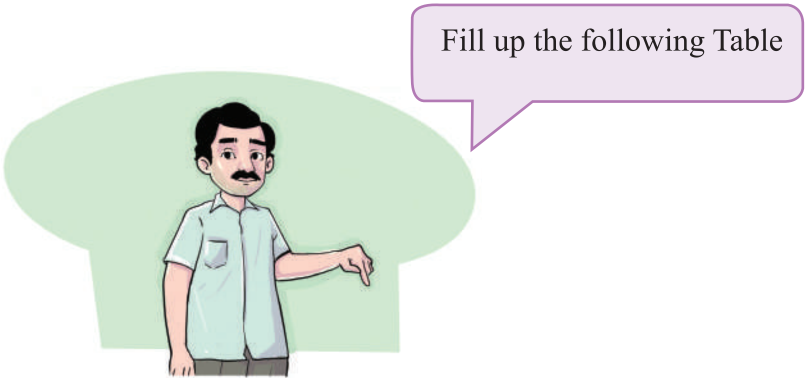
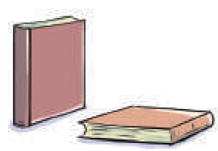
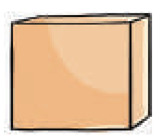
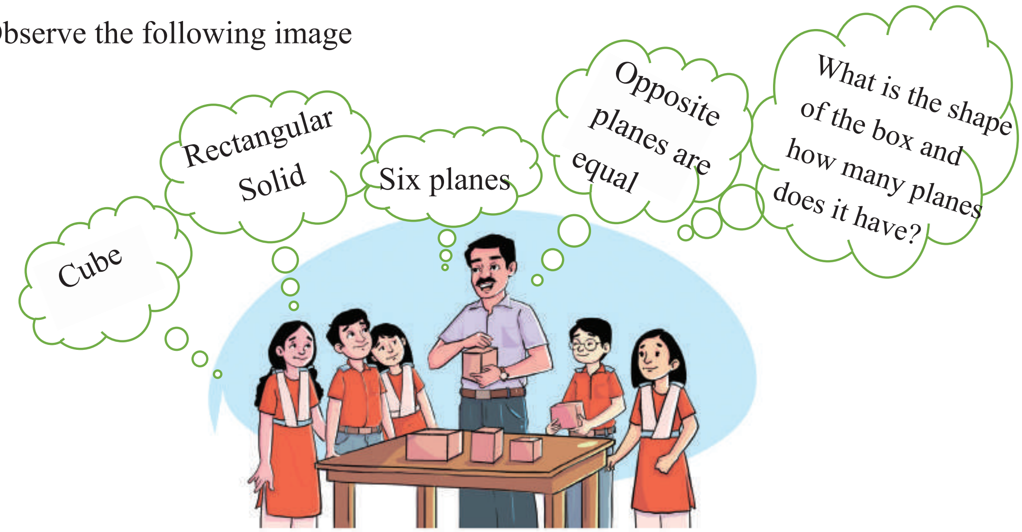


Image	Name	Sides	Angles	Planes	Other charac-teristics (If any)	Name of geometric shape
						
						
						
						
						

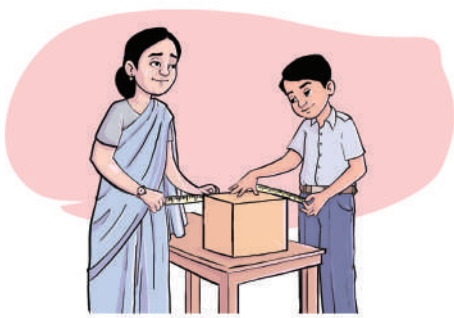
Let's Measure the planes of boxes

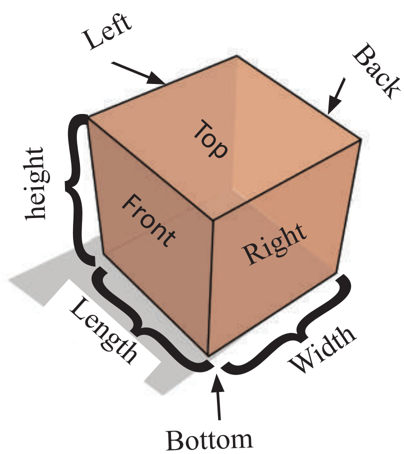
Observe the following image

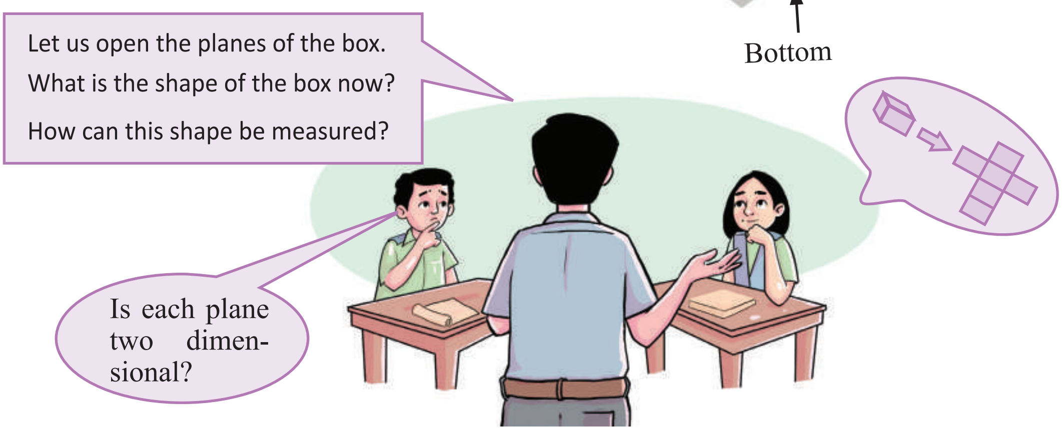


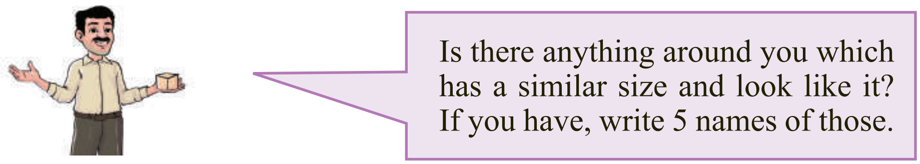
We have learnt different methods to find the area of the planes of two dimensional objects. Now we shall find the areas of all the planes of three dimensional objects in more than one method.

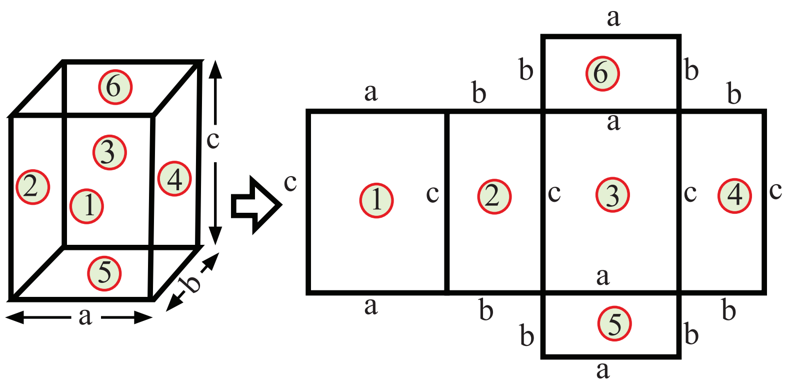
In the following picture, the length, breadth, height and 6 planes of a cube will be identified and shown:











- Is there any relationship between each plane of the box and the plane opposite?
- Without measuring the areas of all the planes, is it possible to find the total surface area of all the planes in any other way?
- Is it possible to measure areas of all the planes of the box by only measuring the areas of the planes numbered 1, 2, 3? If not, then by measuring which three planes, you can find the total surface area of all the planes of the box?

➤ Instructions:

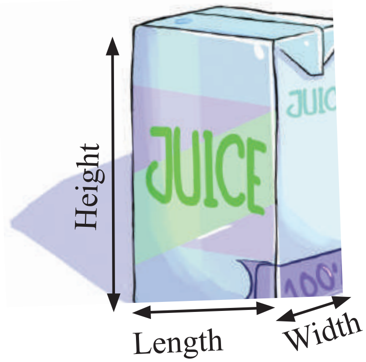
1. Identify the planes.
2. Measure the areas of all the planes and write them down.
3. Find the sum of all the areas.
4. This sum is the total area of your book/ khata/ diary.

Sample of real problems:

Let's take a look at the next picture

Do you know the adverse effects on the environment due to excessive use of paper/ plastic/ polythene?

Individual task : Now look at the following picture:



Collect some packets/boxes in your household and complete the Table below:

Name of product	Length	Breadth	Height	Surface area	Adverse effect on environment (Much/Medium/Little)
Packet of mango juice					
Tissue box					
...					
...					

$\frac{1}{2}$

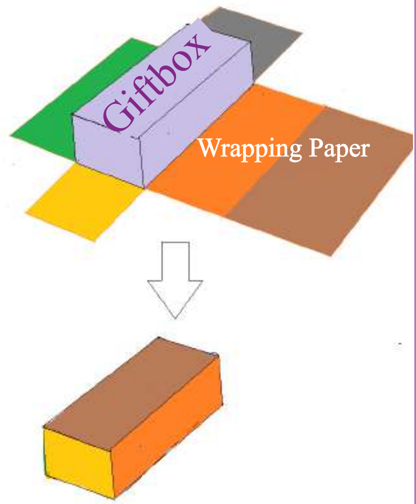
Real life problems:

The length of sides of a cube is 6 cm. Find the total surface area of the object.

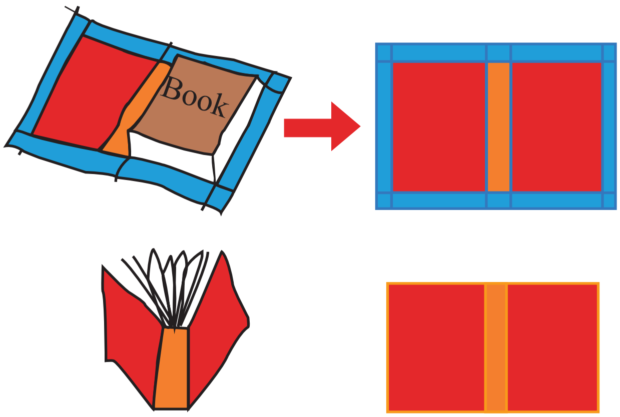
1) The length, breadth and height of a rectangular solid object are 25cm, 20cm and 15 cm respectively. Find the total surface area of this.
2) You want to give a gift to your friend on his/her birthday. Hence you bought a present. The present is kept in a cubic box of length 12 cm. If you want to wrap the box with wrapping paper, what is the minimum size of coloured paper needed?
3) The length of the gift box in the following picture is 24 cm, breadth 12 cm and height 8 cm. How much coloured/white paper will be needed to wrap the box?
4) The length of the gift box in the following picture is 24 cm, breadth 12 cm and height 8 cm. How much coloured/white paper will be needed to wrap the box?

You can present a similar gift box on your friend's birthday, wrapping it with coloured/ white paper.

At list how match paper is needed to wrap the gift ?



5. The length, breadth, and height of the book in the following picture are 10 cm, 6 cm and 4 cm respectively. How much paper will be needed to cover the book, where the width of the blue part around the paper is2 cm?



You may cover your textbooks with coloured/white paper/old calendar pages.

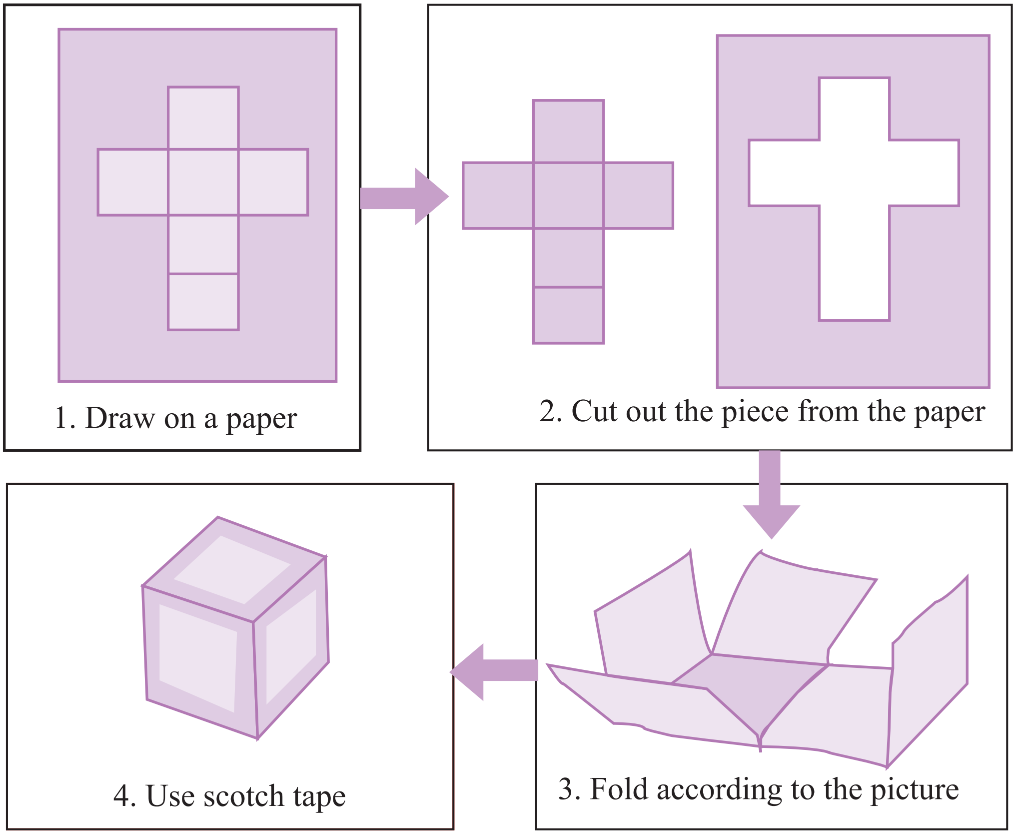
Making three dimensional models and their measurement

Make some similar three dimensional objects and measure them.

Boxes enclosed inside boxes

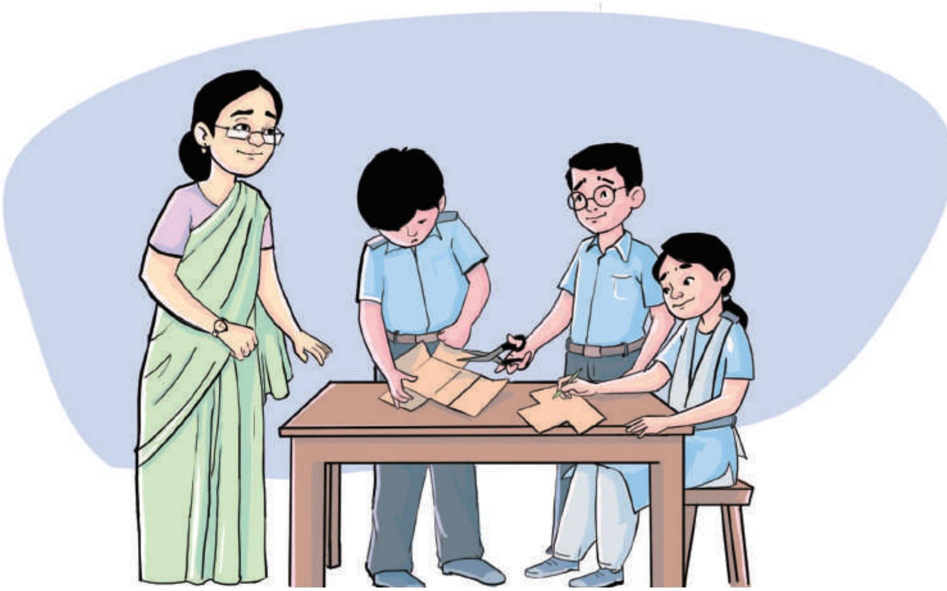
(Measurement of volumes of three dimensional objects)

Making cubic boxes

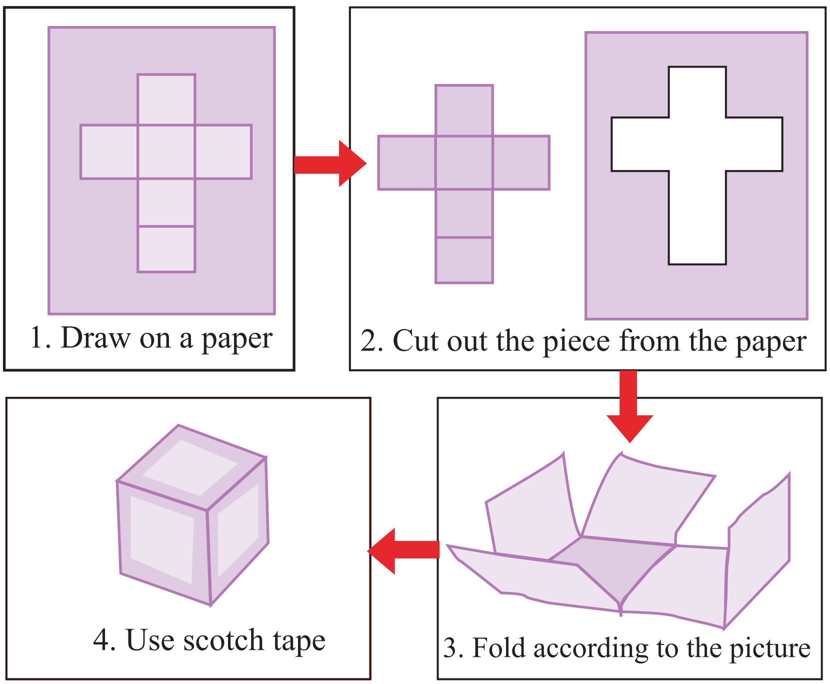


Let us make some cubic boxes by cutting papers according to the instructions given above.

- First take a paper (old calendar page or thick paper), then taking length of fixed units and using ruler and pencil, draw 6 squares (approximately) according to instructions in number (1).
- Then cut the marked part of the paper as in picture number (2) and separate it.
- Next, according to the instructions in number (3), fold the paper to make a box.
- Finally, according to the instructions in number (4), fix the planes of the box with gum or scotch tape to construct the cubic box.



Making box of rectangular size solids

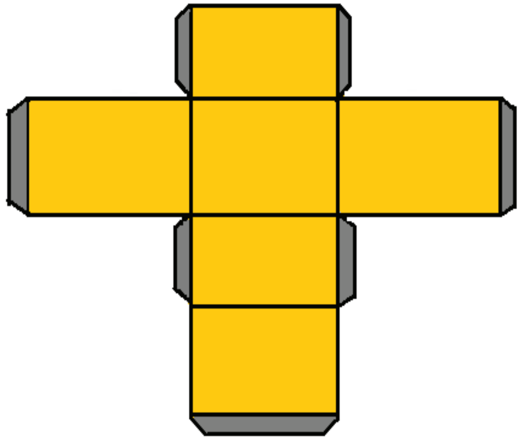


Let us cut some papers like the pictures above and make rectangular solid boxes.

Do you want to glue the planes of the box with each other?

****

Then cut some extra part when cutting the paper as in the picture beside.



Boxes enclosed inside boxes

Many of you must have bought a dozen of match boxes from shops. What are the sizes of the match boxes? All are of same size, aren't they? Keep the twelve small match boxes of same size inside another larger box. Can you tell:

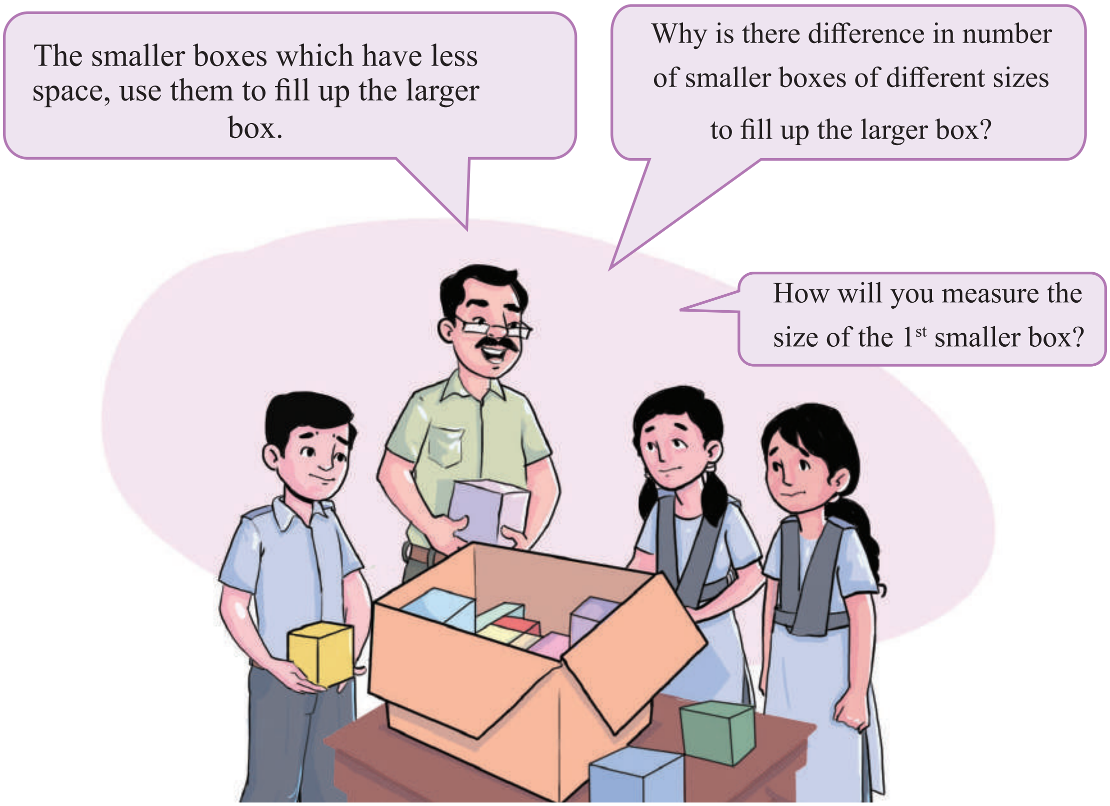
a. Which box was made first, the small box or the larger box?

b. Is there any relationship between the sizes of the small box and the larger box?

- To know the answer, according to the instructions of your teacher, choose a measurement of model of three dimensional cubic and rectangular solid through lottery.
- According to the measurement obtained by lottery, make a model of a three dimensional solid object and measure the areas of the planes and submit it in the next class.
- Now according to the instructions of your teacher, arrange the small boxes made by you to fill up the larger box as shown in the picture.

Fill up the following table by counting the necessary number of smaller boxes to fill up the larger box.

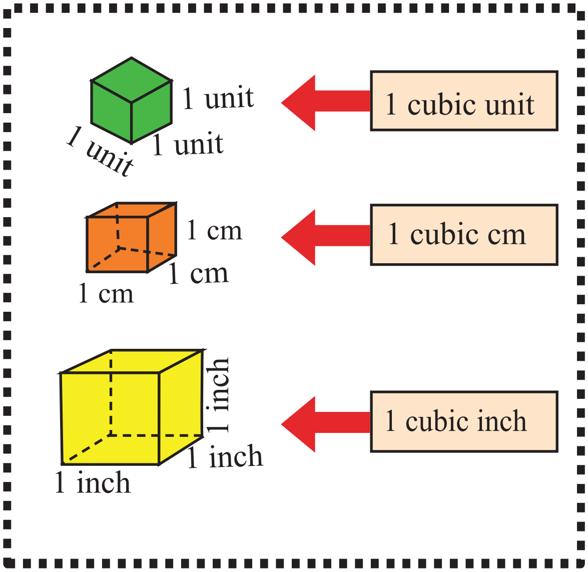
Serial number	shapes of smaller boxes	size of of smaller box	number of smaller boxes needed to fill the larger box
1	length=1 inch, breadth=1 inch, height=1 inch	?	?
2	length=1 inch, breadth=1 inch, height =2 inch	?	?
3	length=2 inch, breadth=2 inch, height=1 inch	?	?

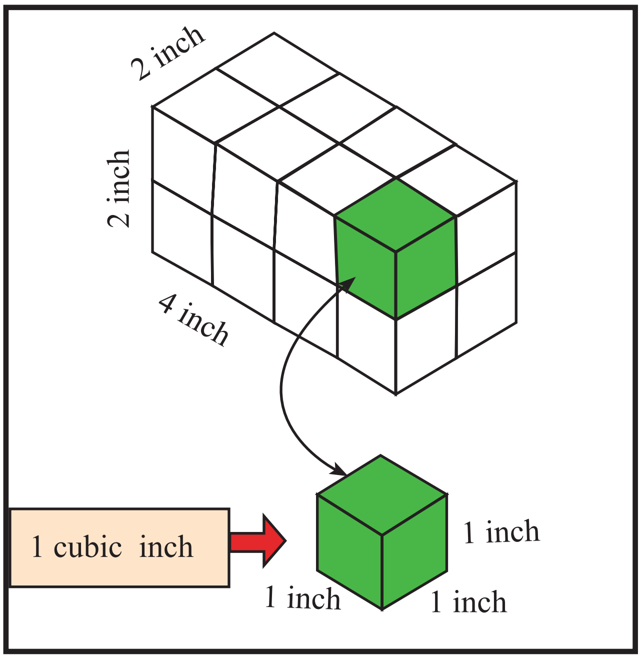


The larger box has space for 16 times the first smaller box.

Volume of the larger box = 16 x volume of 1st smaller box.

Why do we need units to measure volume?



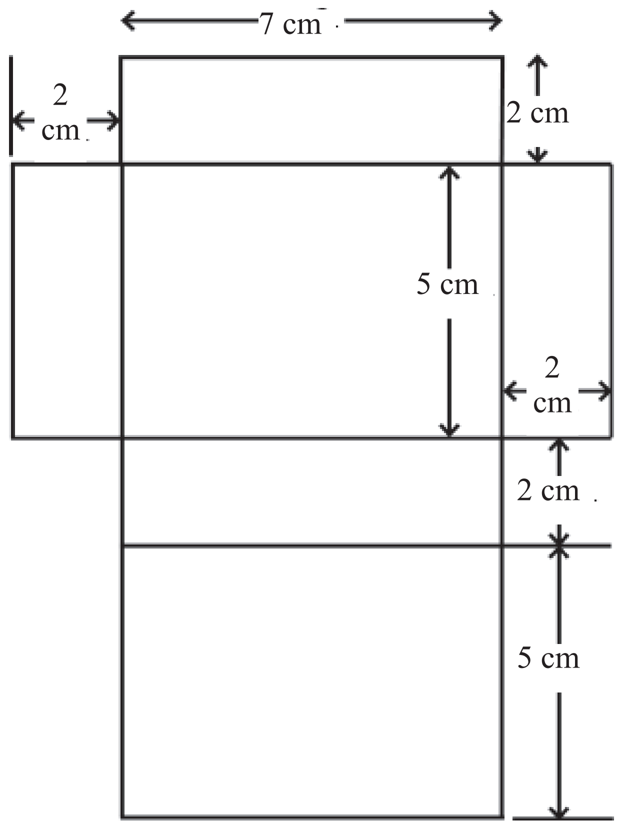


Find the relationship between the volume of the large box with the volumes of the other smaller boxes of two different measurements.

Volume of larger box = 16 x volume of 1st smaller box = 16 x 1 cubic inch = 16 inch^3

Exercise

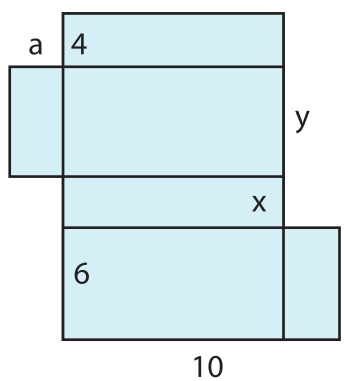
1. Cut out some paper according to the measurement shown in the picture and then fold and attach with scotch tape to form a rectangular solid object. What will be the volume of the rectangular solid object?



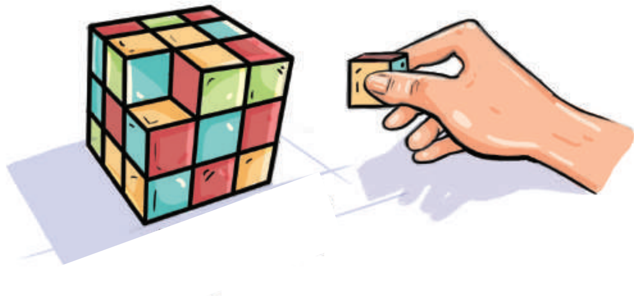
2. The following diagram is an open rectangular box. The measurements are given centimetre unit.

   a) Find values of a, x, y.

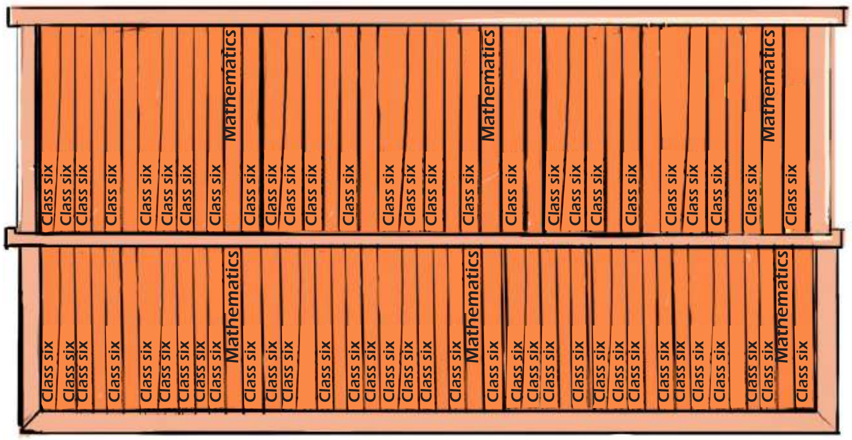
   b) Find the volume of the box.



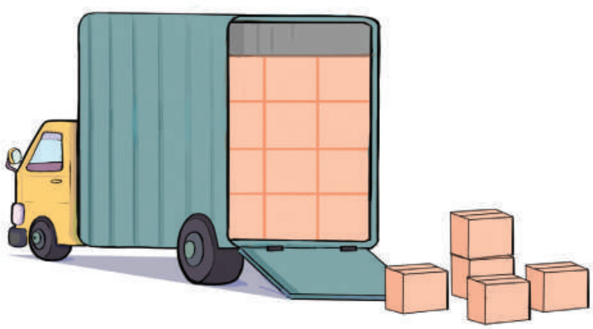
3. How many small cubic size pieces will be needed to make each of the shapes shown in the picture?



4. Find out how many Mathematics Books of class 6 will be needed to fill up one shelf of the Bookshelf of the library of your school?



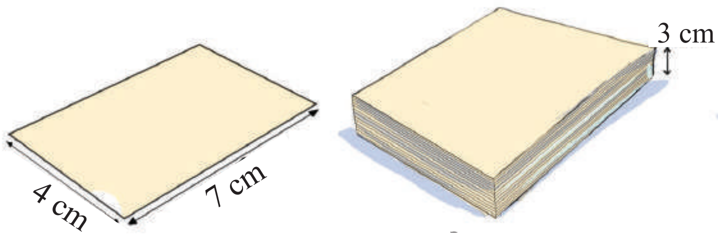
5. A truck has space of 12 feet x 6 feet x 8 feet, to fill up with cartons to carry. If the size of each carton is 2 feet x 2 feet x 1 foot, how many cartons may be possible for the truck to carry?



6. A pile of pages was made by putting 200 pieces of papers, like the one in the picture, with one on top of another.

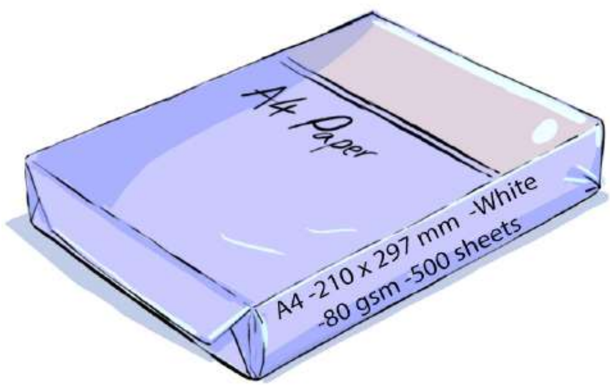
   a) What will be the volume of the pile of pages?

   b) What is the thickness of one page?



1 piece of paper   200 pieces of papers

7. A packet of A4 size papers is seen in the following picture:



Observe what's written on the packet and according to that, fill up the following Table. You may take help from your teacher if necessary.

Length of one Page (mm)	Width of one page (mm)	Colour of paper	Weight of paper per square metre (in gm)	Number of pages per packet

Now answer the following questions:

   a) What is the weight of 1 page?

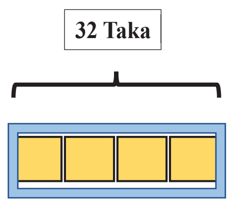
   b) What is the weight of the whole packet?

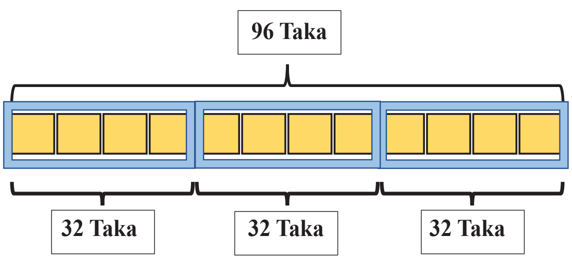
   c) By measuring what is the height of the packet, can you find the thickness of one page?

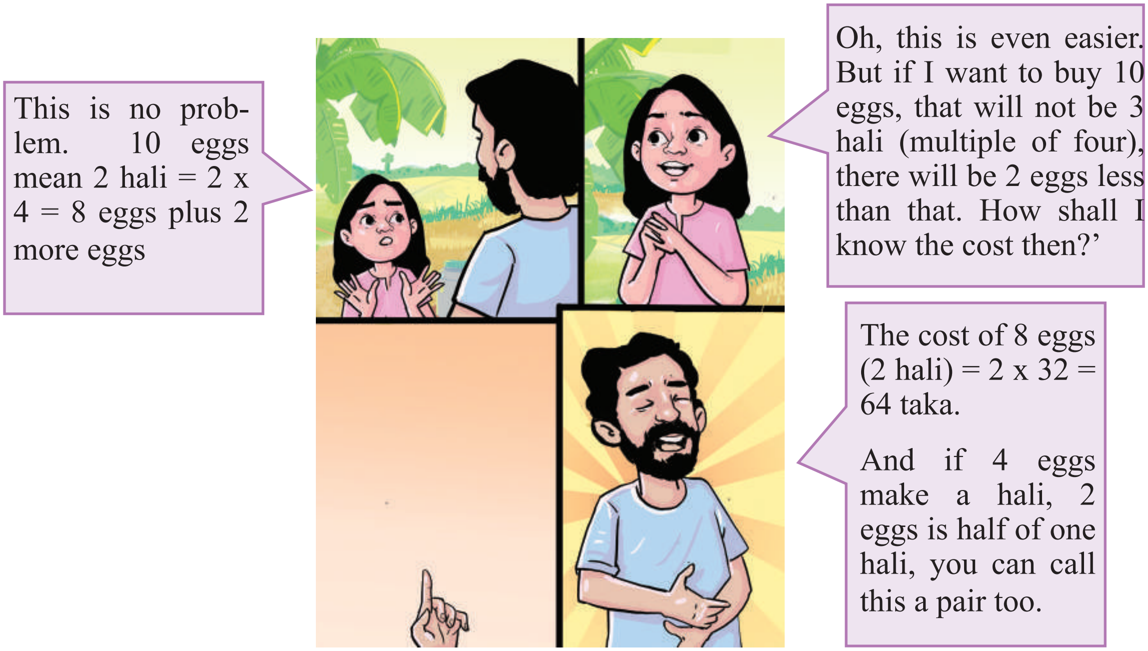
Unitary Method, Percentages and Ratio

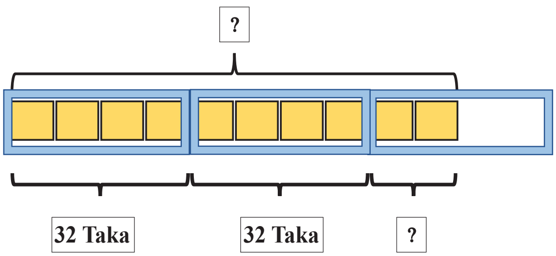
Unitary Method

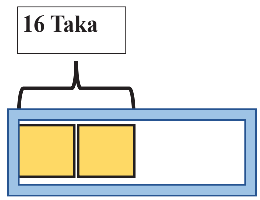
A day in an egg shop



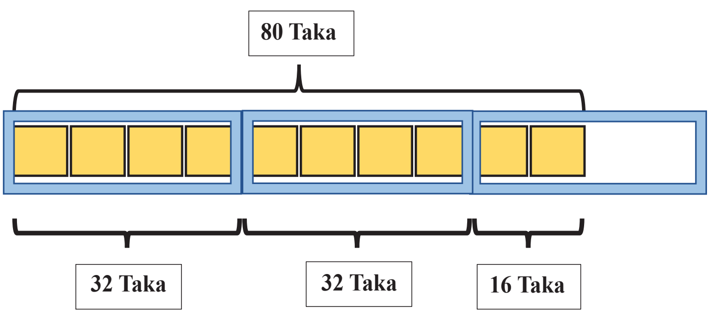




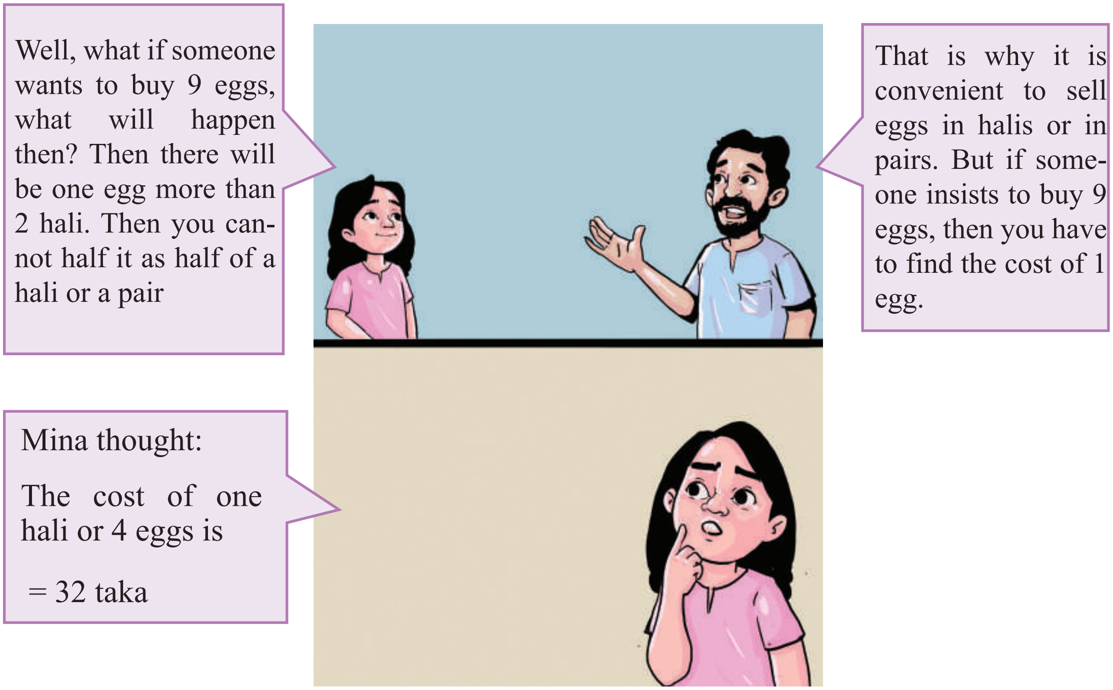


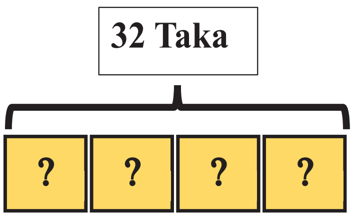


And the cost of 2 eggs will be half the cost of one hali, i.e.32 2 = 16 taka. Now, very easily, you can add the cost of half a hali to the cost of 2 hali to get the cost of 10 eggs = 64 + 16 = 80 taka.



Mina liked the method of the shopkeeper very much. But she still had a question in mind.





Then, the cost of 1 egg = 32 4 = 8 taka

Then, the cost of 9 eggs

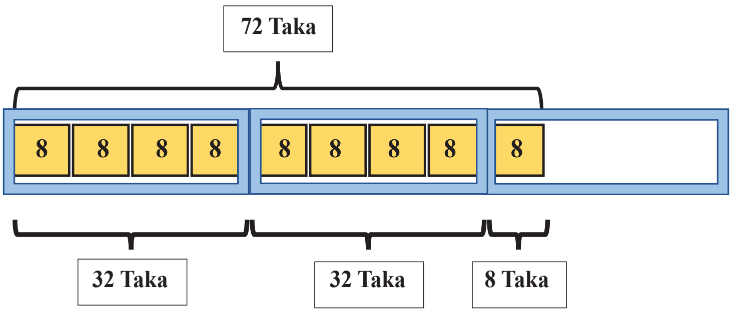
= cost of (2 x 4 + 1) eggs

= cost of 2 hali eggs + cost of 1 egg

= 2 x 32 taka + 8 taka

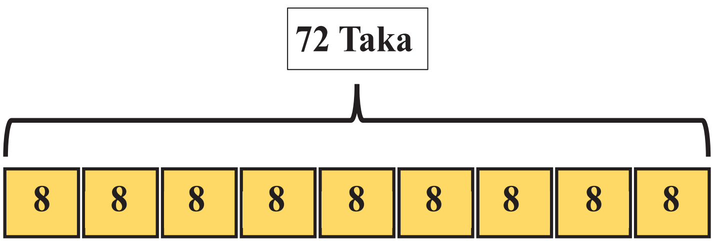
= 64 taka + 8 taka

= 72 taka



Now, Mina noticed something interesting. If the cost of 1 egg is known, then you do not need to know the cost of one hali etc. You just multiply that cost directly by the number of eggs required.

Example: the cost of 9 eggs = 9 x cost of 1 egg = 9 x 8 taka = 72 taka



Now there is an interesting task for you. Complete the task according to the fol lowing steps and write down the full details of the task in your exercise books with pictures and show them to your teacher in the next class.

- Count the total number of eggs consumed by all in your home in a month. Take help from your guardian if necessary.
- Now go to a shop in your area and ask what the cost of a dozen of eggs is. Can you find out what was the cost of eggs for that month for you, without using paper-pencil?
- After returning home, find out the cost using paper-pencil and pictures and determine if the cost figured out while you were in the shop was correct.
- Figure out the total expenses for buying eggs for your family throughout the year from the expense of that one month.
- What sort of problems do you think you may face to compute the total expense for the whole year if the cost of eggs is not same every month?

Painting a Wall

- 6 people want to paint a wall

Here you have to assume that each person can paint the same area of the wall per day.

Now look at the diagram below, how 6 people can paint the complete wall.

Man 1 (Day 1)	Man 1 (Day 2)	Man 1 (Day 3)	Man 1 (Day 4)	Man 1 (Day 5)	Man 1 (Day 6)	Man 1 (Day 7	Man 1 (Day 8)	Man 1
(Man 2 (Day 1)	Man 2 (Day 2)	Man 2 (Day 3)	Man 2 (Day 4)	Man 2 (Day 5)	Man 2 (Day 6)	Man 2 (Day 7)	Man 2 (Day 8)	(Day 9)
Man 3 (Day 1)	Man 3 (Day 2)	Man 3 (Day 3)	Man 3 (Day 4)	Man 3 (Day 5)	Man 3 (Day 6)	Man 3 (Day 7)	Man 3 (Day 8)	Man 3 (Day 9)
Man 4 (Day 1)	Man 4 (Day 2)	Man 4 (Day 3)	Man 4 (Day 4)	Man 4 (Day 5)	Man 4 (Day 6)	Man 4 (Day 7)	Man 4 (Day 8)	Man 4 (Day 9)
Man 5 (Day 1)	Man 5 (Day 2)	Man 5 (Day 3)	Man 5 (Day 4)	Man 5 (Day 5)	Man 5 (Day 6)	Man 5 (Day 7)	Man 5 (Day 8)	Man 5 (Day 9)
Man 6 (Day 1)	Man 6 (Day 2)	Man 6 (Day 3)	Man 6 (Day 4)	Man 6 (Day 5)	Man 6 (Day 6)	Man 6 (Day 7)	Man 6 (Day 8)	Man 6 (Day 9)

So, they can complete painting the wall in 9 days.

■ Now, think about how much time will one person take to paint the whole wall?

You can guess it will take a very long time. But exactly how long? You can find in the diagram.

Man 1 (Day 1)	Man 1 (Day 7)	Man 1 (Day 13)	Man 1 (Day 19)	Man 1 (Day 25)	Man 1 (Day 31)	Man 1 (Day 37)	Man 1 (Day 43)	Man 1 (Day 49)
Man 1 (Day 2)	Man 1 (Day 8)	Man 1 (Day 14)	Man 1 (Day 20)	Man 1 (Day 26)	Man 1 (Day 32)	Man 1 (Day 38)	Man 1 (Day 44)	Man 1 (Day 50)
Man 1 (Day 3)	Man 1 (Day 9)	Man 1 (Day 15)	Man 1 (Day 21)	Man 1 (Day 27)	Man 1 (Day 33)	Man 1 (Day 39)	Man 1 (Day 45)	Man 1 (Day 51)
Man 1 (Day 4)	Man 1 (Day 10)	Man 1 (Day 16)	Man 1 (Day 22)	Man 1 (Day 28)	Man 1 (Day 34)	Man 1 (Day 40)	Man 1 (Day 46)	Man 1 (Day 52)
Man 1 (Day 5)	Man 1 (Day 11)	Man 1 (Day 17)	Man 1 (Day 23)	Man 1 (Day 29)	Man 1 (Day 35)	Man 1 (Day 41)	Man 1 (Day 47)	Man 1 (Day 53)
Man 1 (Day 6)	Man 1 (Day 12)	Man 1 (Day 18)	Man 1 (Day 24)	Man 1 (Day 30)	Man 1 (Day 36)	Man 1 (Day 42)	Man 1 (Day 48)	Man 1 (Day 54)

It is seen in the table that if 1 person is painting the whole wall, then he is doing the work alone for 6 persons. Hence it has taken 6 times more time.

1 person painted the whole wall in = 9×6 or 54 days.

Here, total time needed for 1 person to paint the whole wall is obtained by multiplying the time taken by 6 people to complete painting the whole wall by 9.

■ Now, how much time will be needed if 3 people are told to paint the whole wall?

Surely, it will take less time than completed by 1 person. But exactly how much less time will be needed, find that from the table.

Man 1 (Day 1)	Man 1 (Day 2)	Man 1 (Day 3)	Man 1 (Day 4)	Man 1 (Day 5)	Man 1 (Day 6)	Man 1 (Day 7)	Man 1 (Day 8)	Man 1 (Day 9)
Man 2 (Day 1)	Man 2 (Day 2)	Man 2 (Day 3)	Man 2 (Day 4)	Man 2 (Day 5)	Man 2 (Day 6)	Man 2 (Day 7)	Man 2 (Day 8)	Man 2 (Day 9)
Man 3 (Day 1)	Man 3 (Day 2)	Man 3 (Day 3)	Man 3 (Day 4)	Man 3 (Day 5)	Man 3 (Day 6)	Man 3 (Day 7)	Man 3 (Day 8)	Man 3 (Day 9)
Man 1 (Day 10)	Man 1 (Day 11)	Man 1 (Day 12)	Man 1 (Day 13)	Man 1 (Day 14)	Man 1 (Day 15)	Man 1 (Day 16)	Man 1 (Day 17)	Man 1 (Day 18)
Man 2 (Day 10)	Man 2 (Day 11)	Man 2 (Day 12)	Man 2 (Day 13)	Man 2 (Day 14)	Man 2 (Day 15)	Man 2 (Day 16)	Man 2 (Day 17)	Man 2 (Day 18)
Man 3 (Day 10)	Man 3 (Day 11)	Man 3 (Day 12)	Man 3 (Day 13)	Man 3 (Day 14)	Man 3 (Day 15)	Man 3 (Day 16)	Man 3 (Day 17)	Man 3 (Day 18)

It can be seen in the picture that, when people are painting the whole wall, then the work of 1 person is divided amongst 3 persons. Hence the time required is also one third of one person.

That is, time taken for 3 people to complete painting the whole wall = $\frac{54}{3}$ days = 18 days.

Here, the time taken to complete painting the whole wall by one person is divided by 3 to obtain the time required for 3 persons.

If the number of people is reduced, the number of days to complete the work increases. Again, if the number of people increases, the number of days decreases.

Food Problems

- **In a hostel, there is food for 4 days for 50 students. How many days can 20 students be fed with that amount of food?**

Here we assume that each student may eat the same amount of food each day.

Now see from the image, how the 50 students can eat all the stored food in the hostel.

Student 1 Day 1	Student 1 Day 2	Student 1 Day 3	Student 1 Day 4
Student 2 Day 1	Student 2 Day 2	Student 3 Day 3	Student 4 Day 4
Student 3 Day 1	Student 3 Day 2	Student 3 Day 3	Student 3 Day 4
Student 4 Day 1	Student 4 Day 2	Student 4 Day 3	Student 4 Day 4
...	...	...	...
...	...	...	...
...	...	...	...
Student 48 Day 1	Student 48 Day 2	Student 48 Day 3	Student 48 Day 4
Student 49 Day 1	Student 49 Day 2	Student 49 Day 3	Student 49 Day 4
Student 50 Day 1	Student 50 Day 2	Student 50 Day 3	Student 50 Day 4

Now think in how many days, only one student can consume that amount of food.

He/she will be eating all the food alone, so he/she can eat for a longer period. Look at the picture, how many days will it be.

Student 1- Day 1	Student 1- Day 51	Student 1- Day 101	Student 1- Day 151
Student 1- Day 2	Student 1- Day 52	Student 1- Day 102	Student 1- Day 152
Student 1- Day 3	Student 1- Day 53	Student 1- Day 103	Student 1- Day 153
Student 1- Day 4	Student 1- Day 54	Student 1- Day 104	Student 1- Day 154
...	...	...	...
...	...	...	...
Student 1- Day 48	Student 1- Day 98	Student 1- Day 148	Student 1- Day 198
Student 1- Day 49	Student 1- Day 99	Student 1- Day 149	Student 1- Day 199
Student 1- Day 50	Student 1- Day 100	Student 1- Day 150	Student 1- Day 200

That means, 1 person can eat the same amount of food 50 times more days.

Hence 1 student has food for 200 or 4 x 50 days.

Remember, 20 students will have to eat the food that one student can eat for 200 days.

Now find out from the table, how many days can 20 students eat and fill up the (empty) spaces.

Student 1- Day 1	Student 1- Day 2	Student 1- Day 3	Student 1- Day 4	...	...	Student 1- Day -☐
Student 2- Day 1	Student 2- Day 2	Student 2- Day 3	Student 2- Day 4	...	...	Student 4- Day ☐
Student 3- Day 1)	Student 3- Day 2	Student 3- Day 3	Student 3- Day 4	...	...	Student 3- Day ☐
Student 4- Day 1	Student 4- Day 2	Student 4- Day 3	Student 4- Day 4	...	...	Student 4- Day ☐
...	...	...	...	...	...	...
...	...	...	...	...	...	...
...	...	...	...	...	...	...
Student 18- Day 1	Student 18- Day 2	Student 18- Day 3	Student 18- Day 4	...	...	Student 18- Day -☐
Student 19- Day 1	Student 19- Day 2	Student 19- Day 3	Student 19- Day 4	...	...	Student 19- Day ☐
Student 20- Day 1	Student 20- Day 2	Student 20- Day 3	Student 20- Day 4	...	...	Student 20- Day ☐

A little attention will tell you that, **20 students will be able to eat for one twentieth of days 1 student can eat. Here 20 students will share the food of 1 student.**

Hence 20 students have food for = $\frac{50\times4}{20} = \frac{200}{20} = 10$ days.

If the number of students decreases, the same amount of food may be eaten for more days and if the number of students increases, number of days decreases.

Did you understand when it is multiplied and when it is divided by, in the Unitary Method?

Now solve the following real life problems using pictures

1) If the cost of 7 kg rice is Tk 280, what is the cost of 15 kg rice?

2) A hostel has food stored for 15 days for 50 students. In how many days can 25 students eat the same amount of food?

3) Shafiq walks 10 hours daily and can travel 480 km in 12 days. How many days will he take to travel 360km in the same pace?

4) 6 persons can cut the crops of a field in 28 days. How many days will 24 persons take to cut the crop of that field?

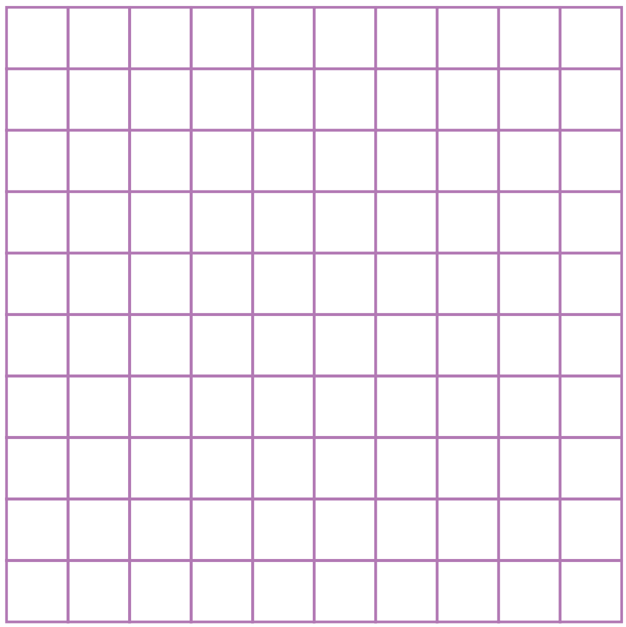
- Find an incident that happens in your real life around you and that you can solve applying the unitary method.
- Then you write the problem and describe the process of solution and draw a picture of it on a poster paper and present it in front of your teacher and classmates in the next class.

Percentage

Percentages in Hundred grids

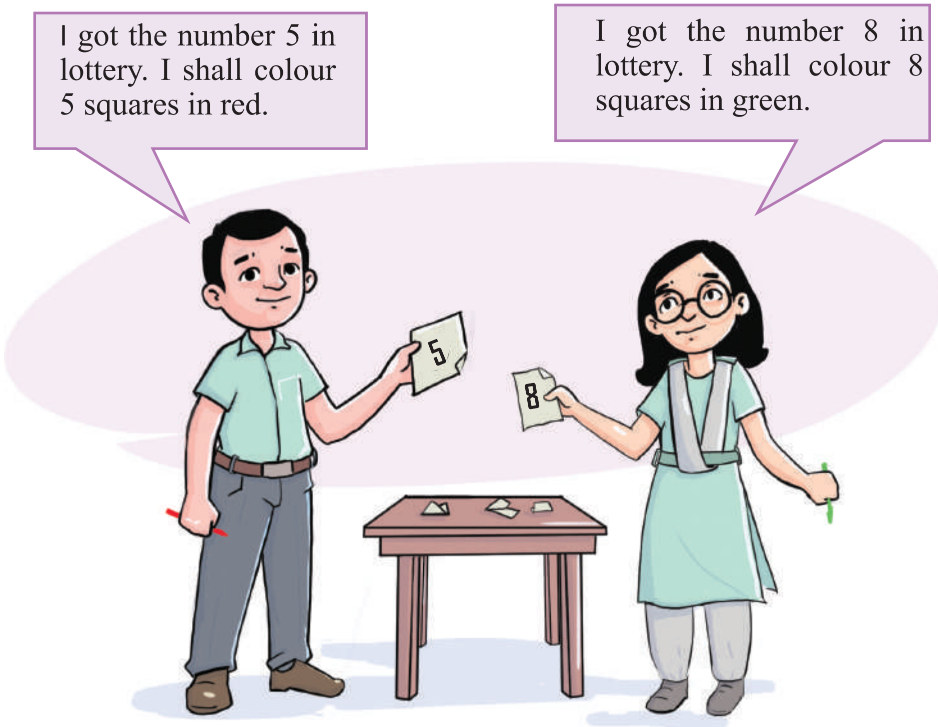
Materials:

- Appropriate number of A4 size papers (each with 100 square grids)
- Appropriate number of small pieces of papers with 1 – 10 written on them
- Appropriate number of colour pencils (of two colours)
  - Today we shall play an interesting game. The game will be in pairs.
  - For each pair, take one A4 size paper and draw a grid of 100 squares as in the picture below. Take help from your teacher if necessary.

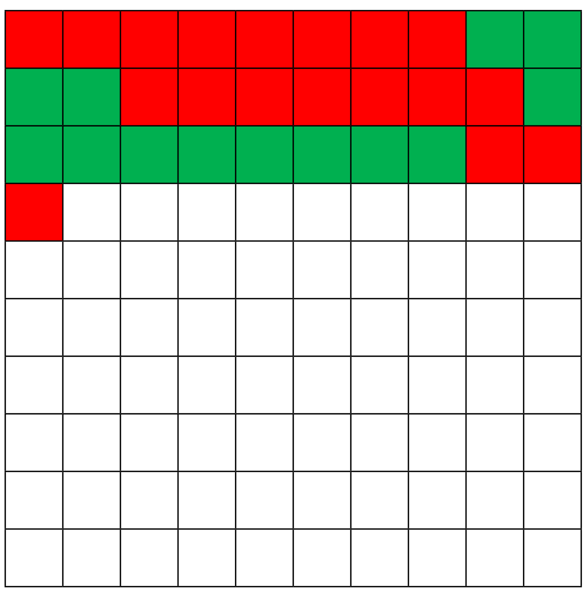


Grid of 100 squares

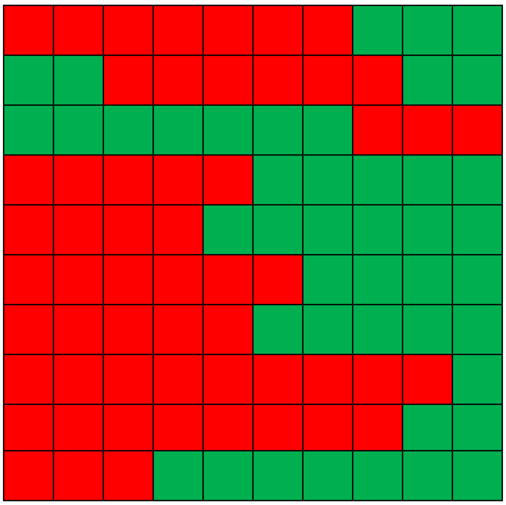
- Each pair of students takes colouring pencil of two colours in your hand.
- Each pair makes 10 small pieces of papers writing 1 – 10 on them.
- Fold these 10 pieces of papers and draw a lottery. The student who gets whichever number in the lottery, he/she will colour that many squares of the grid with the colour pencil in hand.



- Then draw another lottery. This time also, similarly, colour the squares according to the number each obtained. Carry on colouring through lottery, until all the squares are coloured.



- At the end stage, a student must get the same number through the lottery, as the number of empty squares is left in the grid. Only then he/she can fill up with the colour pencil. If that number is not obtained, then must draw the lottery again.
- There were 100 squares in total. Count the number of squares coloured by your own colour pencil, and find out who coloured how many squares?
- Be aware that the sum of the squares coloured by both must be 100. Hence find out who painted how many out of the 100.
- The student, whose number of coloured squares is more, is the winner.



Total squares	First student of the pair	Second student of the pair
100	56	44

- There were 100 squares in total. The 1^{st} student of the pair coloured 56 squares out of 100, and the 2^{nd} student of the pair did 44 out of 100.
- We can write the matter as follows-
- The 1^{st} student of the pair coloured 56 out of 100 or $\frac{56}{100}$ part or or 56%.
- The 2^{nd}student of the pair coloured 44 out of 100 or $\frac{44}{100}$ part or 44%

- You must be thinking, what symbol is this? %

- This is the symbol of Percentage.

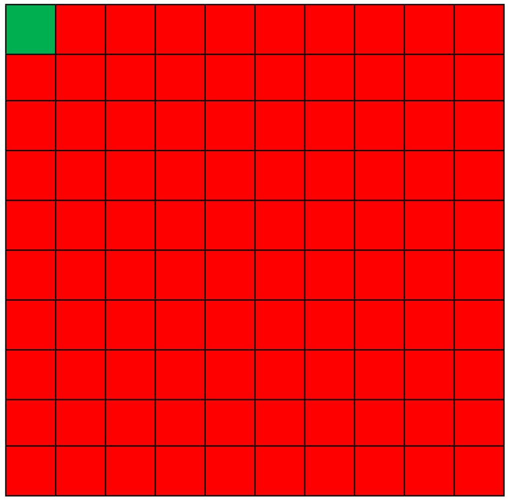
'Percentage is such a fraction whose denominator is 100'
The name 'percent' explains that this is related to hundred or 100.

You can see from the example above, if the denominator of a fraction is 100, then the value of the numerator what part of 100 it is and that is the percentage.

Again, the sign % means 1 part out of 100 or $\frac{1}{100}$.

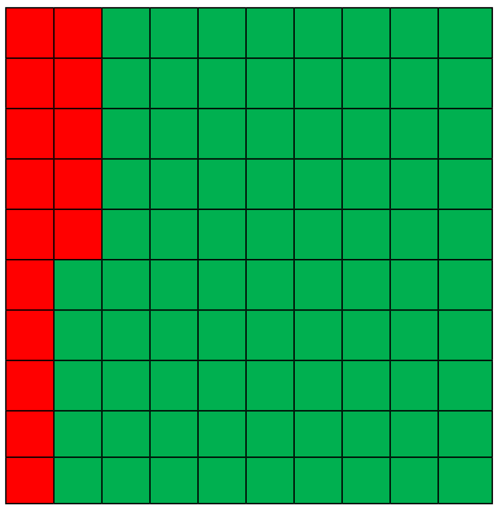
In the picture beside, % or $\frac{1}{100}$ is shown using green colour.

Again, this sign can also express $1\% = \frac{1}{100}$.

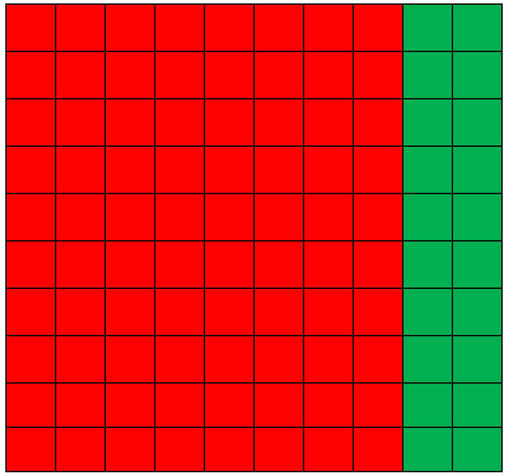


- You will understand the meaning of the percentage symbol and its use, from the following examples:

$15\% = \frac{15}{100}$



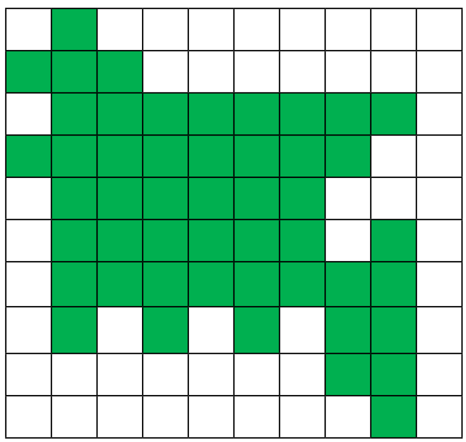
$80\% = \frac{80}{100}$



Individual Task : Now solve the following problems:

1. (a) Here, what is the percentage of the green coloured part?

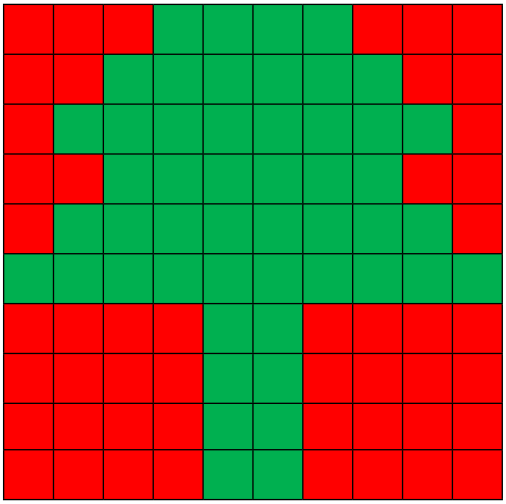
Green coloured = ☐ %



(b) What is the name of the green coloured shape? Have you seen such shape before?

Your Answer :

2. What is the percentage of green coloured part and of red coloured part, out of the whole part in the following pictures?



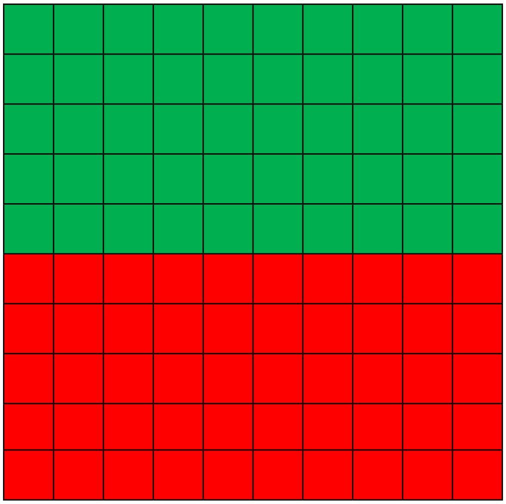
a) Green coloured part = ☐ %

Red coloured part = ☐ %

b) What is the name of the green coloured shape? Have you seen such shape before?

Your Answer : ☐

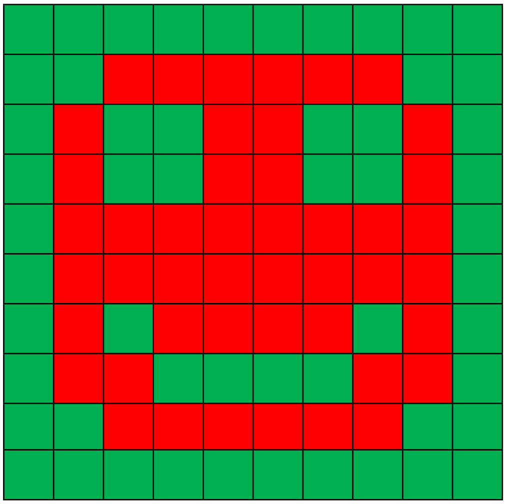
c)



Green coloured part = ☐ %

Red coloured part = ☐ %

d)

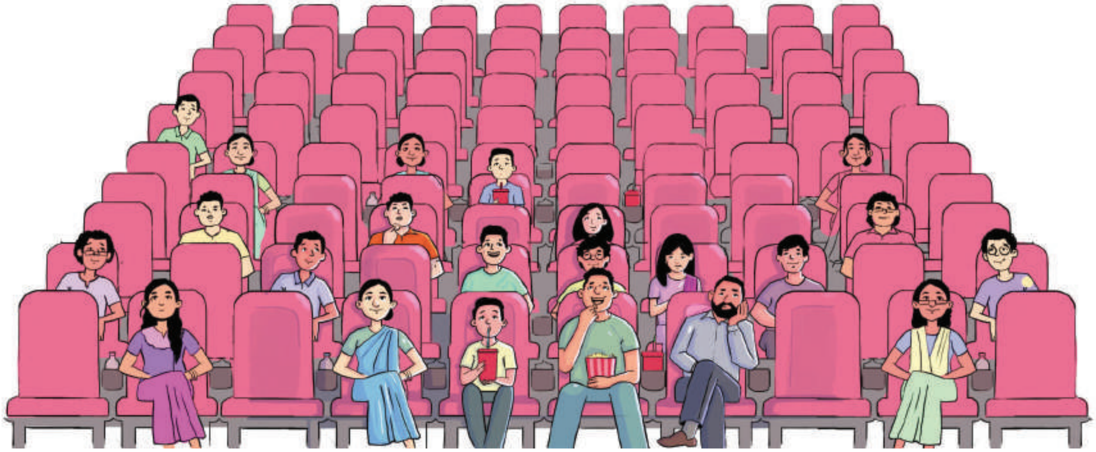


Your answer:

Green coloured part = [ ] %

Red coloured part = [ ] %

3. What percentage of the gallery is full and what percentage is empty in the following image?



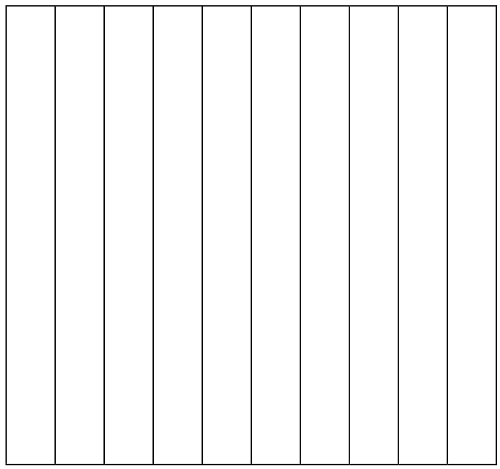
Your answer,

Full part = [ ] %

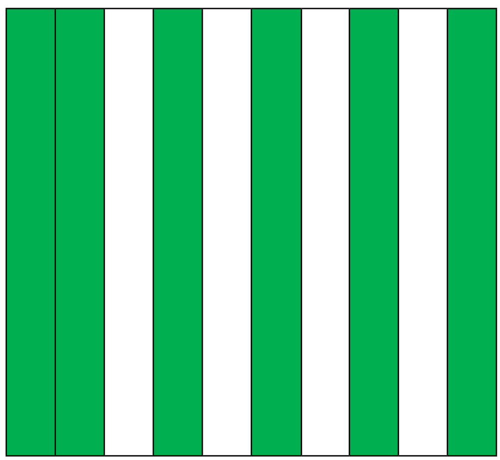
Empty part = [ ] %

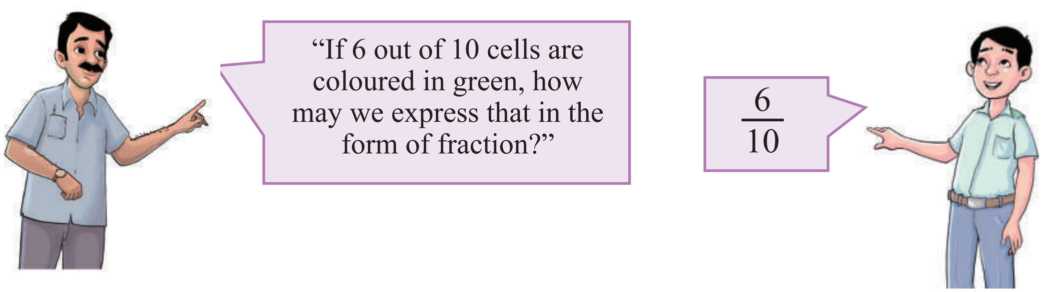
Relationship between Fractions and Percentages

- Draw a graph of 10 columns like the picture, in your exercise books.

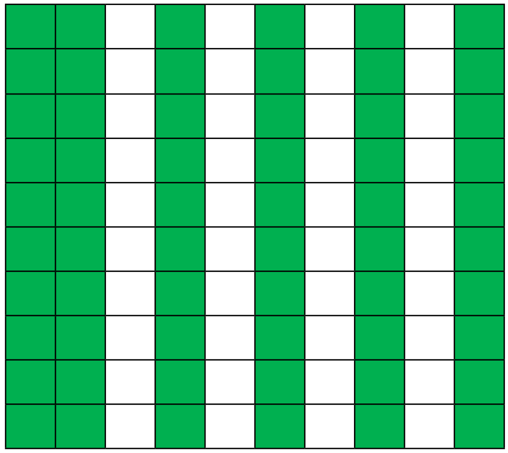


- Now, colour any 6 of the columns of the graph. Use green colour.





Now how can we express $\frac{6}{10}$ as a percentage?



In that case, we have to make the denominator 100. How is that possible?

Divide each of the 10 rectangles of the graph into 10 parts to get total of 100 parts.

Now see in the image and count that there are 60 green parts out of 100.

Therefore, coloured 6 parts out of 10 means $\frac{6}{10}$ or $\frac{60}{100}$ are coloured or 60% are coloured.

Observe, according to the technique above, dividing each of the 10 rectangles of the graph into 10 parts and according to the idea of equivalent fractions, multiplying the denominator and numerator by 10, means the same thing.

In this case also we get the same result: $\frac{6 \times 10}{10 \times 10} = \frac{60}{100} = 60\%$

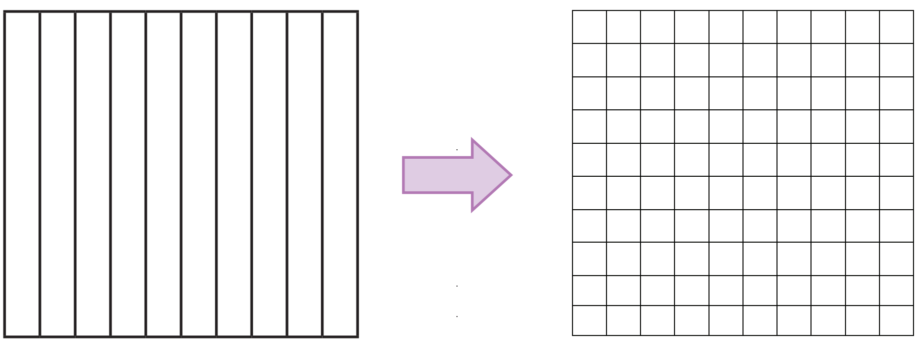
Again $\frac{6}{10}$ part of 100 parts = $100 \times \frac{6}{10} = 10 \times 6 = 60$ parts of the graph.

In this way also we can transform $\frac{6}{10}$ into percentage.

Now solve the following percentage related problems.

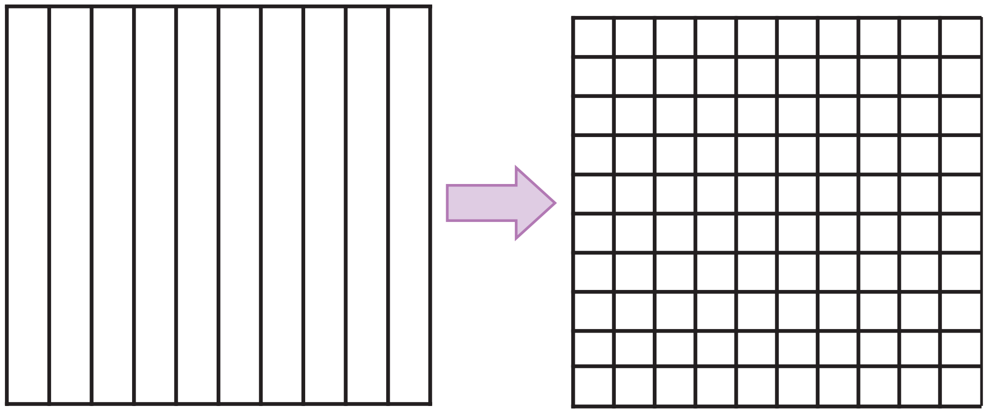
1. Use green pencil and express the following fractions as percentages with colour:

a. $\frac{3}{10}$



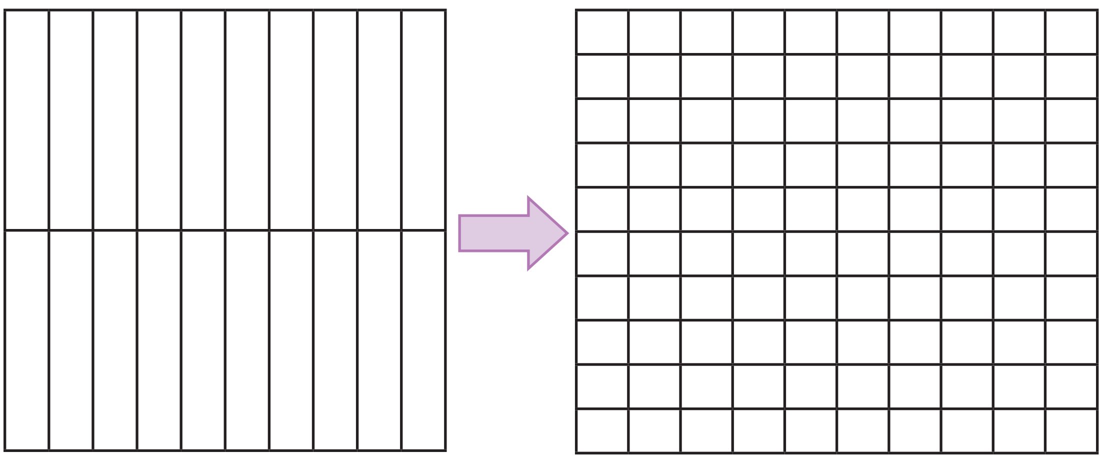
$$\frac{3}{10} = \frac{\square}{100} = \square\,\%$$

b) $\frac{1}{2}$



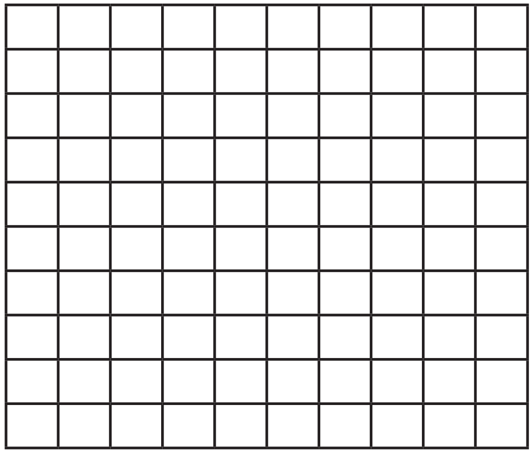
$$\frac{1}{2} = \frac{\square}{10} = \frac{\square}{100} = \square\%$$

c) $\frac{1}{4}$



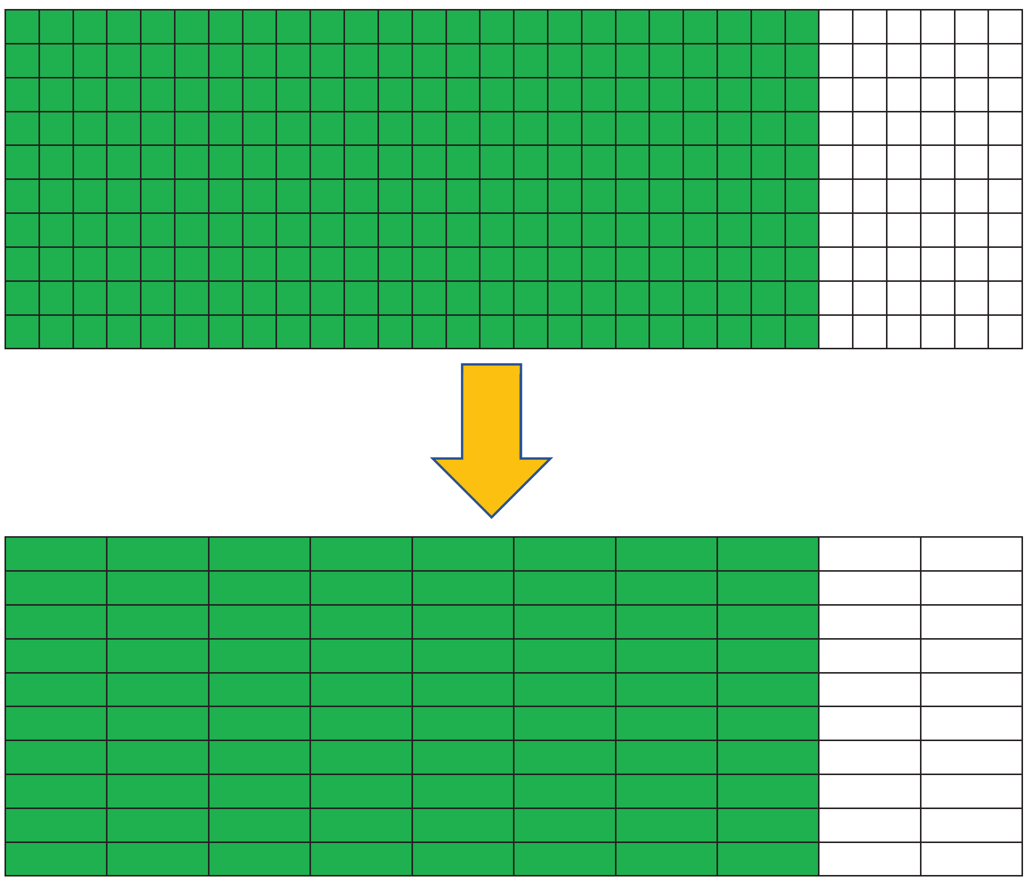
$$\frac{1}{4} = \frac{\square}{20} = \frac{\square}{100} = \square \text{ again, } \frac{1}{4} = \frac{1 \times \square}{4 \times \square} = \frac{\square}{100} = \square\ \%$$

d) $\frac{7}{25}$



$\frac{7}{25} = \frac{\square}{100}$ or, $100 \times \frac{7}{25} = \square$ Hence, $\frac{7}{25} = \frac{\square}{100} = \square\%$

2. You obtained 240 marks out of 300 in some examination. Then what is the percentage of your marks out of the total mark?

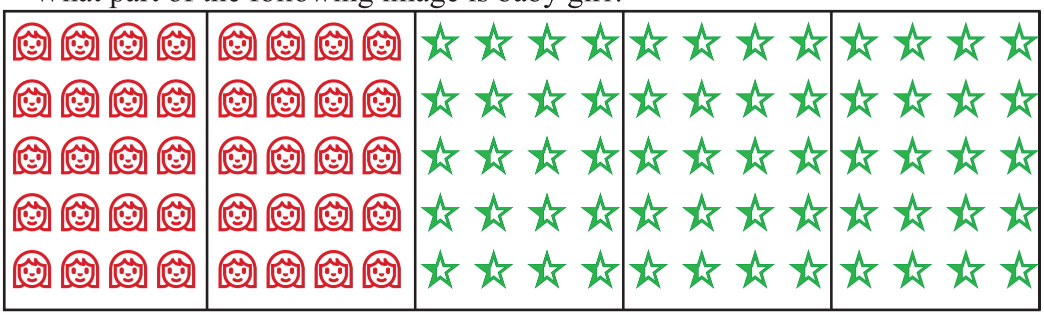


Expressing the fraction in the simplest form, we get: $\frac{240}{300} = \frac{\square}{100} = \square\,\%$

Again, marks obtained out of 100 = $100 \times \frac{240}{300}\,\% = \square\,\%$

3) In the picture, part of a wall is painted. Then what percentage of the wall is painted?

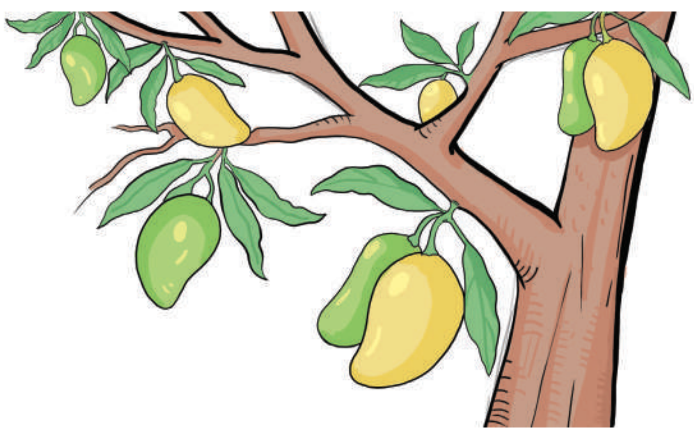
4) What part of the following image is baby girl?



Baby girl within the total image is = $\square\,\% = \frac{\square}{100} = \frac{\square}{\square}$

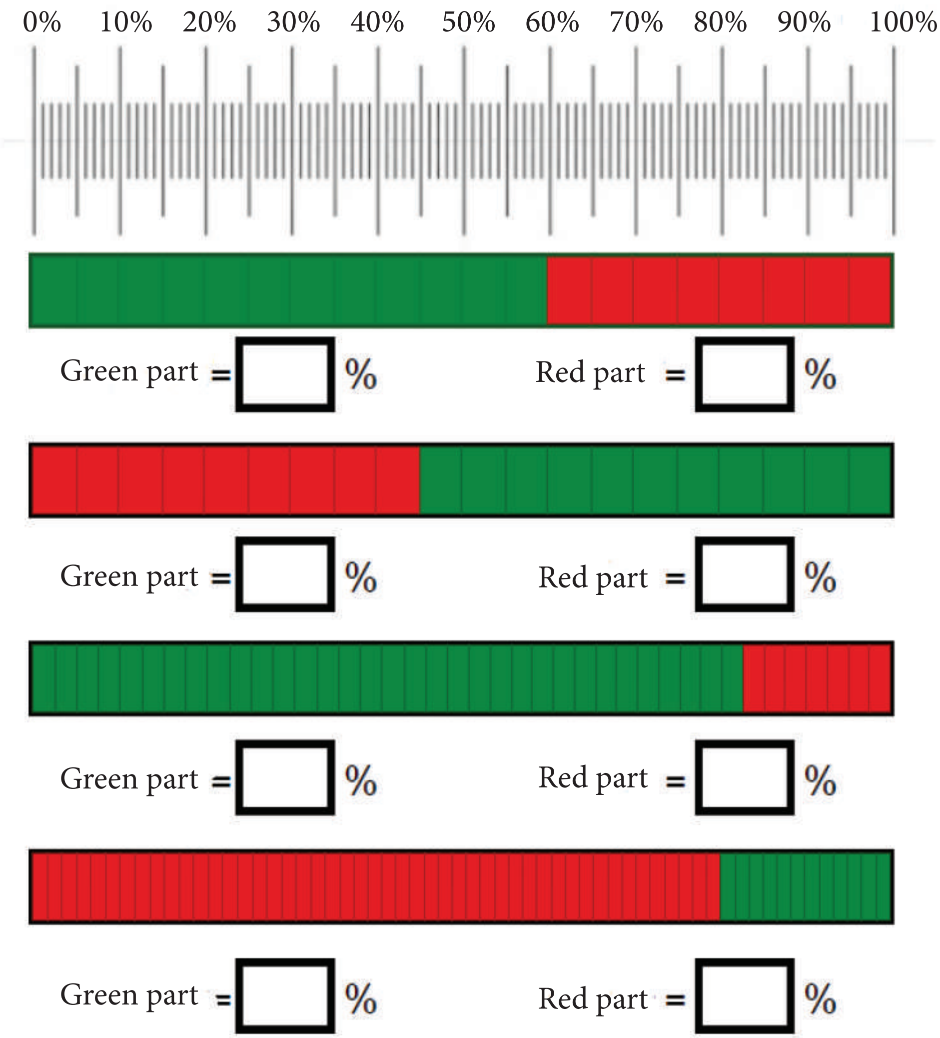
Then, out of total image $\frac{\square}{\square}$ part is baby girl.

5) What is the percentage of green/raw mangoes in the picture below?



Percentage in Bar Model

Use the scale shown in picture. Find what percentage of the bars is coloured green and what percentage is coloured red:



Now let us listen to a story.

Tisha's journey to Sylhet

Tisha boarded a bus from Khulna to go to Sylhet, with 2500 taka in hand. She paid 800takas for the bus fare.

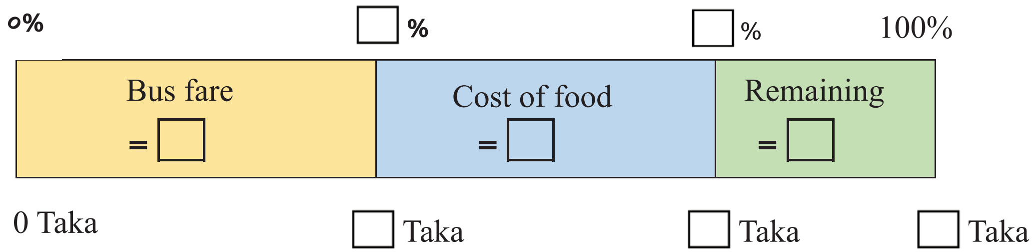
When the bus stopped on the way, Tisha bought some food.

After reaching Sylhet, Tisha found that 80% of her total money is already spent. Now, can you answer the following?

- What is the percentage of the bus fare out of the total amount Tisha had?
- What is the total expenditure of Tisha?
- How much money Tisha was left with?
- How much did Tisha spend on food?
- What is the percentage of money spent on food out of the total money?
- What is the percentage of money spent on food out of the total spent?

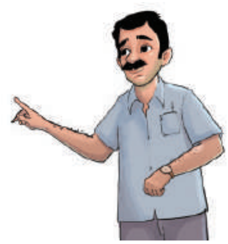
"You may use the bar graph model to use the idea of percentages."

"Let's put all the information obtained from the beginning of the story, in a bar model."



'But you could not fill up the empty box just from the information of the story.'

'Now you can find all the answers using the bar model and the empty boxes will be filled too.'



Bus fare out of total money $= \dfrac{\text{bus fare}}{\text{total money}} \times 100\% = \dfrac{\square}{\square} \times 100\% = \square\ \%$

- Amount of money Tisha spent out of the total $\square\ \% = \square \times \dfrac{\square}{100} = \square$ Taka
- So, money left with Tisha = Total money – Total spent = $\square - \square = \square$ Taka

  If you want, you can find the remaining money Tisha has, using percentage too.

  The remaining money from total $= 100\% - \square\ \% = \square\ \%$

Hence, the remaining money is $\square$ % of toal = $\square$ of $\square$ % = $\square \times \square = \square$ Taka

- Cost of Tisha's food = total expense – bus fare = $\square - \square = \square$ Taka

- Cost of food out of total money = $\frac{\text{cost of food}}{\text{total money}} \times 100\% = \frac{\square}{\square} \times 100\% = \square\ \%$

  If you want, you can find the percentage of the cost of food out of the total money, without finding the actual amount of cost of food.

  Percentage of cost of food = percentage of total expenses – percentage of bus fare.

  That means, cost of food out of total amount $= \square\% \quad \square\% = \square\%$

- Cost of food out of total expense = $\frac{\text{cost of food}}{\text{total money}} \times 100\% = \frac{\square}{\square} \times 100\% = \square\%$

  If you want, you can find the percentage of cost of food, from the percentage of total expense, without finding the total expense.

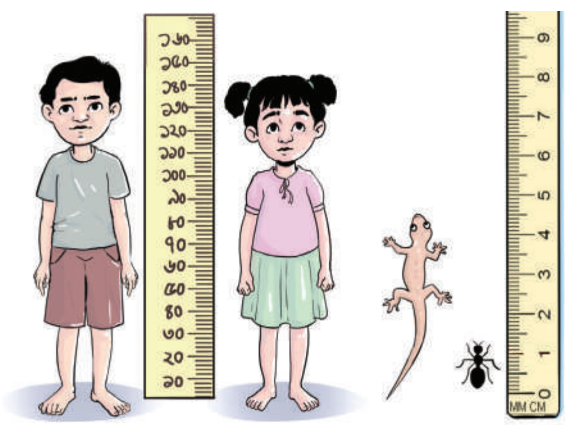
Cost of food out of total money = $\square\% = \text{Total taka} \times \frac{\square}{100\%}$

And total expenditure out of total money = $\square\% = \text{Total taka} \times \frac{\square}{100}$

Then, cost food out of total cost $= \left(\frac{\text{Total taka} \times \frac{\square}{100}}{\text{Total taka} \times \frac{\square}{100}}\right)\% = \left(\frac{\frac{\square}{100}}{\frac{\square}{100}}\right)\% = \frac{\square}{\square}\% = \square\%$

Ratio

We often compare two similar objects in our daily life. For example, suppose the height of Nabil is 150 cm and the height of his sister, Nova is 143 cm. Now can you suggest how you can compare the heights of the two? One way is, find the difference by subtracting. That is, the height of Nabil is (150 – 143) cm or 7 cm more than his sister Nova. Now let us compare the lengths of a lizard and an ant. Suppose length of a lizard is 8 cm and length of an ant is 1 cm. Here also the difference of the lengths of lizard and ant is (8-1) cm or 7 cm.



It can be seen here, difference of heights of Nabil and Nova and, lizard and ant are the same. But the idea you get from 'the difference of heights of Nabil and Nova is 7 cm'; is same as 'the idea of difference of lengths of lizard and ant being 7 cm'; then how much appropriate will it be? Think about it!

You will rather get a better idea if you find how many ants lined up one after another to be equal to the length of a lizard.

If you divide the length of a lizard by the length of an ant, you will get $= \frac{8}{1} = 8.$

That means, putting 8 ants one after another, will make the length equal to length of a lizard.

You can also say, length of lizard is 8 times the length of ant, or, a lizard is 8 times longer than an ant.

Comparing substances through division, how many times bigger or smaller, is known as Ratio.

Mathematical symbol for ratio is ':'

Mathematically it is written, ratio of lengths of lizard and ant = 8:1.

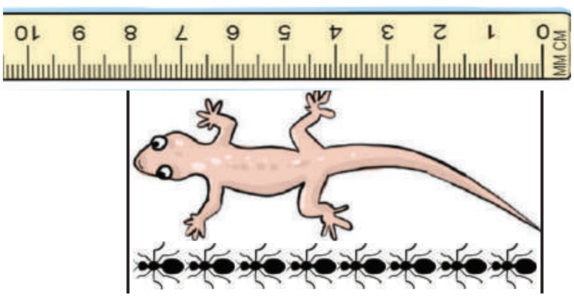
Again, if you divide the length of ant by length of lizard you will get: $\frac{1}{8}$.

That is the length of ant is equal to one eighth of the length of lizard. Again, you may also say, an ant is 8 times smaller compared to the length of lizard.

Mathematically written, ratio of lengths of ant and lizard = 1 : 8

Therefore, ratio is really a fraction.

You can understand it better from the picture, what a ratio is. 30



- ❖ Find similar some incidents where it is easier to compare through division or ratio instead of finding differences.
- ❖ For each of the incidents, objects being compared, find their difference and ratio, both.
- ❖ Why is it easier to compare through ratio, give your reasons.
- ❖ What do you understand by ratio for each incident, show it by drawing pictures (Can draw similar pictures as above, of ratio of lengths of lizard and ant.

Let us now solve real life problems using ratio.

- Weight of Shawkat is 30 kg and weight of his father is 60 kg. How many times more is the weight of father than that of Shawkat?

Ratio of the weights of father and Shawkat is:

$= \frac{60}{30}$

$= \frac{2}{1}$ (dividing the numerator and denominator by 30) = 2 :

Here, weight of father is $\frac{2}{1}$ of Shawkat or 2 times more.

Collect the information for your class and fill up the empty boxes.

Number of male students $= \square$

Number of male students $= \square$

Total number of students $= \square$

- ❖ Ratio of male – female students

  $= \frac{\square}{\square} = \frac{\square}{\square}$ (dividing numerator and denominator by $\square$) $= \square : \square$

- ❖ Ratio of number of male students and total number of students

  $= \frac{\square}{\square} = \frac{\square}{\square}$ (dividing numerator and denominator by $\square$) $= \square : \square$

- ❖ Ratio of number of female students and total number of students

  $= \frac{\square}{\square} = \frac{\square}{\square}$ (dividing numerator and denominator by $\square$) $= \square : \square$

- ❖ Ratio of total number of students and male students

  $= \frac{\square}{\square} = \frac{\square}{\square}$ (dividing numerator and denominator by $\square$) $= \square : \square$

- ❖ Ratio of total number of students and female students

  $= \frac{\square}{\square} = \frac{\square}{\square}$ (dividing numerator and denominator by $\square$) $= \square : \square$

➢ All the rectangles below are of same length.

1 unit

Ratio of the lengths of green coloured part and yellow coloured part $= \frac{\square}{\square} = \square : \square$

Ratio of the lengths of yellow coloured part and green coloured part $= \frac{\square}{\square} = \square : \square$

Ratio of the lengths of green coloured part and length of rectangular part $= \frac{\square}{\square} = \square : \square$

Ratio Ratio of the lengths of yellow coloured part and length of rectangular part $= \frac{\square}{\square} = \square : \square$

Rafique bought 6 packets of red pens and 2 packets of blue pens from the shop.
Ratio of packets of red pens and packets of blue pens

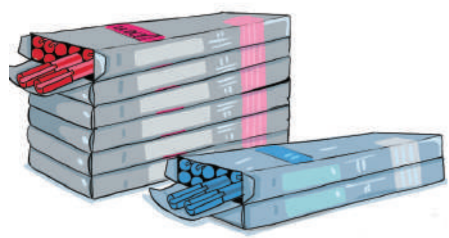
$= \frac{\square}{\square}$

$= \frac{\square}{\square}$ (dividing numerator and denominator by $\square$) $= \square : \square$

Each packet of red and blue pen has10 pens.

So Rafique bought red pens $= 6 \times \square = \square$

And blue pens $= 2 \times \square = \square$



Ratio of number of red pens and blue pens

$= \frac{\square}{\square}$

$= \frac{\square}{\square}$ (dividing numerator and denominator by $\square$) $= \square : \square$

Is the ratio of packets of red pen and blue pen same as the ratio of numbers of red pens and blue pens? $\square$ yes $\square$ no

Monika bought 6 packets of red pens and 2 packets of blue pens from the shop.
Ratio of packets of red pens and packets of blue pens

$= \frac{\square}{\square}$

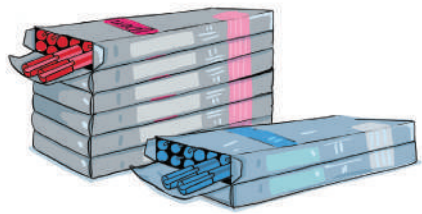
$= \frac{\square}{\square}$ (dividing numerator and denominator by $\square$) $= \square : \square$

Each packet of red pen has10 pens. Each packet of blue pen has 12 pens.

So, Monika bought red pens. $= 6 \times \square = \square$

And blue pens $= 2 \times \square = \square$

Ratio of number of red pens and blue pens $= \dfrac{\square}{\square}$



$= \dfrac{\square}{\square}$ (dividing numerator and denominator by $\square$)

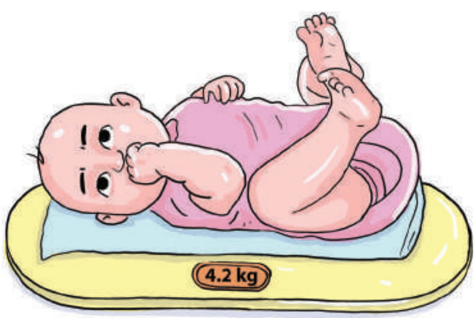
Are ratio of packets of red pens and blue pens and ratio of number of red pens and blue pens same? $\square$ yes $\square$ no

If each packet of red and blue pens has the same number of pens, you can find the ratio of number of pens from the ratio of number of packets. But that is not possible if the packets of red and blue pens have different number of pens.

❖ Ratio of the weight of the baby and fish show in the picture $= \dfrac{\square}{\square}$

$= \dfrac{\square}{\square}$ (dividing numerator and denominator by $\square$)

$= \square : \square$



Now think, is it possible to compare the age of a baby with the weight of another baby? Never! In case of comparisons, the two objects/subjects must be of the same type.

- Let us consider again, age of the brother is 3 years and age of the sister is 6 months. What will be the ratio of their ages?

  Here, two similar quantities, the ages of brother and sister is being compared. Be careful, the age of brother is more than the age of sister. That is, the brother is older than the sister.

  Now if we compare directly, without considering the units, do you know what will happen?

  Ratio of the ages of brother and sister = $\frac{3}{6} = \frac{1}{2} = 1:2$.

  Then the matter will be more like the age of the brother is = $\frac{1}{2}$ of the age of sister, that is half the age.

  But is it really so? The age of brother is certainly not less than the age of the sister and 3 years is definitely not half of 6 months. Surely something is wrong in the computations.

  Observe that, in all previous cases we compared two quantities of the same unit. Hence the ratios gave the correct ideas.

  Here, we are not getting the correct ratio, since we are comparing with two units, year and month.

  In this case, though they are similar type of quantity, we cannot directly compare the ages of both. The two quantities to be compared must be of same unit.

  Hence, the ages of both need to be converted either into years or months.

  In this case, we shall convert the ages of both into months.

  Then the age of brother is 3 years= 36 months ($\because$ 1 year = 12 months) and

  The age of sister is 6 months.

  Then the ratio of the ages of brother and sister is = $\frac{36}{6}$

  $= \frac{6}{1}$ (dividing numerator and denominator by 6) = 6 : 1

Suppose a child is 6 years old and another child is 9 years and 6 months old. Then how will you find the ratio of the ages of the two children?

We know that, to find the ratio, the units of both the quantities must be same.

First, convert the ages of both the children into months.

Here, age of the first child = 6 years = $\square$ months

Age of the other child = 9 years 6 months = $\square$ months

$= \frac{\square}{\square}$

$= \frac{\square}{\square}$ (dividing numerator and denominator by $\square$) = $\square : \square$

$= \square : \square$

- Convert the ages of both children into years and find the ratio of their ages.
- Compare the ratio obtained by converting the ages of both children into months.

- How many times or what part of one of two similar quantities is, compared to the other, can be expressed as a fraction. This fraction is known as the ratio of the two quantities.
- But for this comparison, the quantities must be of same type. The quantities are be converted to same type or same.
- The ratio has no unit since it is the quotient of two similar quantities or of same units.

Now solve the problems below according to the rules of Ratios:

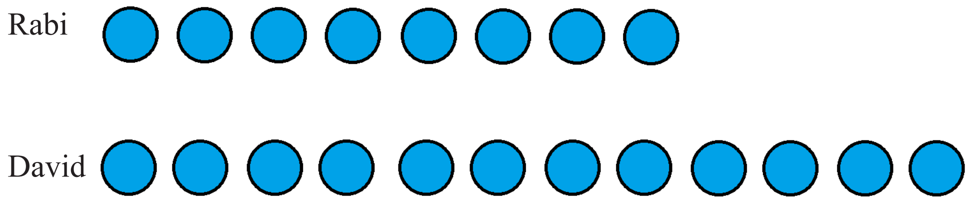
1. Find the ratios of the first quantity to the second quantity of the following pairs of numbers:
   (a) 25 and 335 (b) $7\frac{1}{3}$ *and* $9\frac{2}{5}$ (c) 1.25 and 7.5 (d) $8\frac{2}{3}$ and 0.125
   (e) 1 year 2 months and 7 months (f) 7 kg and 2 kg 300 g (g) tk 2 and 40 poysa
2. Count the number of books and exercise books you brought to class and complete the following tasks:
   (a) Find the ratio of numbers of exercise books and.
   (b) Find the ratio of the total numbers of pages of the exercise books and of the books.

3) Find the length and breadth of your Mathematics book using a ruler and find their ratio.
4) Find 3 different tables in your classroom or home or somewhere else.
   (a) Find the length and breadth of each table and find the ratio amongst them.
   (b) Find for which table the ratio of the length and breadth is the greatest.
5) Do you know any story or incident where the word ‘ratio’ has been used? Or have you seen the word ‘ratio’ or the symbol ‘:’ written anywhere? Find some real incidents and draw pictures or describe how and where you found them and tell your teachers and classmates.

6. Search and find some examples around your real life or you heard about, where same types or similar two quantities have been compared, but their units were different. Then describe how the units were converted to the same unit.

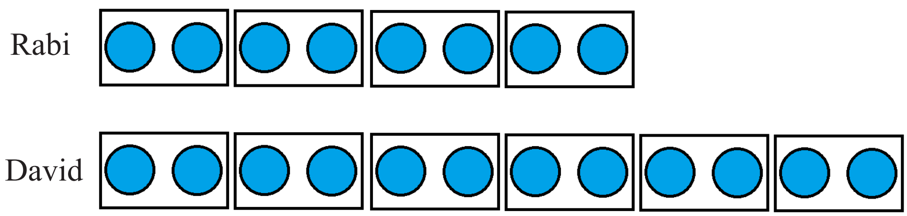
Equivalent Ratios

Rabi has 8 marbles and David has 12 marbles.



Then ratio of numbers of marbles of Rabi and David = 8 : 12

Now Rabi and David made packets of their marbles with two marbles per packet.



Now, number of packets of marbles Rabi has $= \frac{8}{2} = 4$

And number of packets of marbles David has $= \frac{12}{2} = 6$

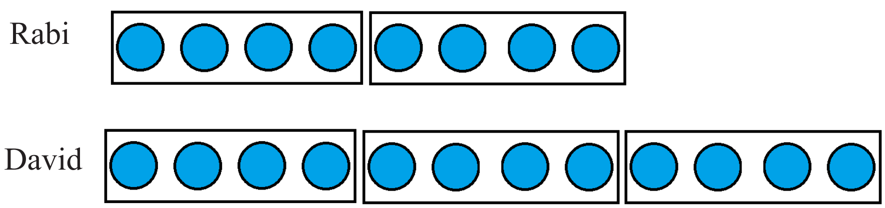
Hence, the ratio of number of packets of Rabi and David = 4 : 6

Since each packet of marbles has the same number of marbles,

Hence the ratio of number of marbles of Rabi and David will be:

$$8 : 12 = \frac{8}{12} = \frac{(8 \div 2) \times 2}{(12 \div 2) \times 2} = \frac{4\ packets \times number\ of\ marbles\ in\ 1\ packet}{6\ packets \times number\ of\ marbles\ in\ 1\ packet} = \frac{4\ packets}{6\ packets} = 4 : 6$$

Now Rabi and David made packets of their marbles with 4 marbles per packet.



Now, number of packets of marbles Rabi has $= \frac{8}{4} = 2$

And number of packets of marble David has = $\frac{12}{4} = 3$

Hence, now the ratio of number of packets of Rabi and David = 2:3

So, now, the ratio of number of marbles of rabi and David will be:

$$8 : 12 = \frac{8}{12} = \frac{(8 \div 4) \times 4}{(12 \div 4) \times 4} = \frac{2\,packets\ \times number\ of\ marbles\ in\ 1\ packet}{3\,packets\ \times number\ of\ marbles\ in\ 1\ packet} = \frac{2\,packets}{3\,packets} = 2 : 3$$

So, observe that, the value of the ratios, 8:12, 4:6 and 2:3 are same. Hence these ratios can be said to be equivalent ratios.

And theratio 2:3 is the simplest form the ratio.

Example: 2:3 = $\frac{2}{3} = \frac{2\times2}{3\times2} = \frac{4}{6}$ = 4:6

$\therefore$ 2 : 3 and 4 : 6 are equivalent ratios

Any ratio has infinitely many equivalent ratios. For example, 2:3, 4 : 6 and 8:12 are equivalent ratios.

Observe:

- The value of a fraction does not change, if the numerator and denominator of a fraction are multiplied by a number other than zero (0).
- Dividing the numerator and denominator of a fraction by their highest common factor, the fraction can be expressed in the simplest form.
- We know that the ratio is a fraction.

If a ratio is converted to a fraction –

- The first term of the ratio is written as the numerator of the fraction and it is called the fore quantity of the ratio.
- The second term of the ratio is written as the denominator of the fraction and is called the after quantity.

Equivalent fractions and equivalent ratios are of same meaning. So, we can say-

- If the fore and after quantity of a ratio is multiplied or divided by a non-zero number, the value of the ratio does not change, and ratios are called equivalent ratios.
- Equivalent ratios can be formed in the same method of forming equivalent fractions.
- A ratio can be simplified by dividing the two quantities of the ratio by their highest common factor.

Now let us solve the following problem involving equivalent fractions.

Fill up the blank:

$10 : 15 = \square : 3 = 6 : \square$

Write the ratios as fractions:

$$\frac{10}{15} = \frac{\square}{3} = \frac{6}{\square}$$

Here the value of the three ratios is same, that is, they are equivalent ratios.

So, we can use the equivalent fractions or equivalent ratios and their characteristics to find the numbers in the empty boxes: $\frac{10}{3 \times 5} = \frac{\square}{3} = \frac{\square \times 5}{3 \times 5}$

That is, $10 = \square \times 5$ or, $\square = \frac{10}{5} = 2$

Again, $\frac{2}{3} = \frac{6}{\square}$ $or, \frac{2}{3} = \frac{2\times3}{\square} = \frac{2\times3}{3\times3}$ $then, \square = 3 \times 3 = 9$

That means, we can write the three equivalent fractions and ratios, including the empty boxes as

$\frac{10}{15} = \frac{\boxed{2}}{3} = \frac{6}{\boxed{9}}$ or, $10 : 15 = \boxed{2} : 3 = 6 : \boxed{9}$

Solve the following problems:

1) Simplify the following ratios:
   (a) 9 : 12 (b) 15 : 21 (c) 45 : 36 (d) 65 : 26

2) Identify the equivalent ratios below:
   12 : 18; 6 : 18; 15 : 10; 3 : 2; 1 : 3; 2 : 6; 12 : 8

3) In a school, there are 450 boys and 500 girls. Write the ratio of the boys and girls of the school in the simplest form.

4) Fill up the empty boxes of the following equivalent ratios: 2
   (a) $2 : 3 = 8 : \square$ (b) $5 : 6 = \square : 36$
   (c) $7 : \square = 42 : 54$ (d) $\square : 9 = 63 : 81$

5) The ratio of the width and length of a hall room is 2 : 5. Fill up the following table with possible values of the width and length:

Width of hallroom (metre)	10		40		160		2.25	$15\frac{3}{5}$
Length of hallroom (metre)	25	50		200		$\frac{3}{4}$		

- Measure the length and width of any three rooms of your school or collect the information with the help of a teacher.
- Find the ratio of the length and width of each room.

Set the Formula, get the Formula

We shall celebrate birth month today. You all know that we celebrate birthday on a day of each month of someone who was born on that month. We all shall eat chocolates today on birthday today. I have 900 chocolates in total. But we shall play a fun game today while distributing the chocolates. The game is- the first person will take 1 chocolate. The 2nd person will take 2 more than the first person. The 3rd person will take 2 more than the 2nd person. In this way, each of next person will be taking 2 more chocolate than the previous person. In our class there are 30 students in all, and I brought chocolate for everyone. Now let me check and count before distribution whether everyone will get a chocolate or not.

1st person got

2nd person got

3rd person got

Distribution of chocolate will continue like this

According to the rule,

Number of chocolate of 1st person = 1 = 1

Total number of chocolates of 1st 2 persons = 1 + 3 = 4 = 2

Total number of chocolates of 1st 3 persons = 1 + 3 + 5 = 9 = 3

Total number of chocolates of 1st 4 persons = 1 + 3 + 5 + 7 = 16 = 4

Total number of chocolates of 1st 5 persons = 1 + 3 + 5 + 7 + 9 = 25 = 5

Distributing the chocolates, it is seen that for the 1st 6 persons, (6) chocolates will be needed, for the 1st 7 persons, (7 chocolates will be needed and so on.

Hence, we can say that total number of chocolates needed for 30 students = $(30 \times 30) = 900$.

That means, if we want, we can distribute the 900 chocolates to everyone, according to the rule of the game.

Hence, we can say that if the number of students is n, then according to rule of game, number of chocolates will be $n \times n$.

Individual task : Distribute 992 chocolates among some people. The first person gets 2, second person gets 2 more than the first, third person gets 2 more than the second person and so on. How many persons will get chocolates in this manner?

Let us cut some papers colour them and make some designs

Cut some rectangular papers, use two different colours of your choice. Then make some designs of paper blocks as the pictures below.

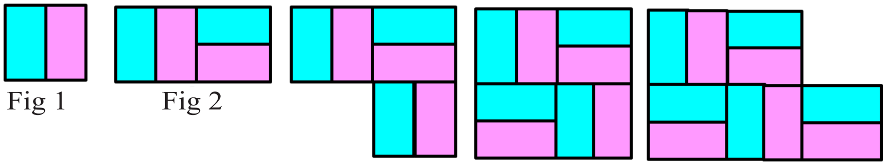
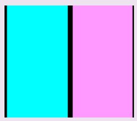
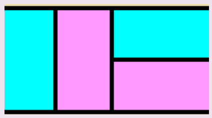
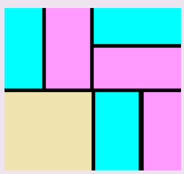


Fig 1 Fig 2 Fig 3 Fig 4 Fig 5

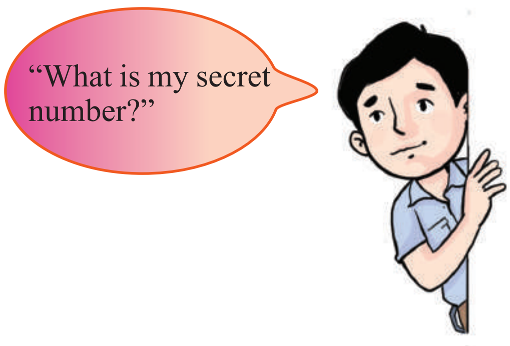
Now fill up the following table:

Figure Number	Figure	Blocks	Number of lines
1st		2	7
2nd		?	?
3rd		?	?
4th			
5th			
6th			
7th			
8th			
9th			

Number of lines of each picture of the table above can be expressed through a Mathematical formula or rule. Write the Mathematical formula or rule using abstract quantities and give logical explanation. Similarly, if you want to create the 50^{th} picture, find what will be the number of blocks and lines.

To find the mystery of a secret number

Ishan and Bindu are playing two similar interesting games. The game is– Ishan thought of a whole number between 1 and 100. Ishan gave some hints to find the secret number. You have to find the secret number by analysing the hints.

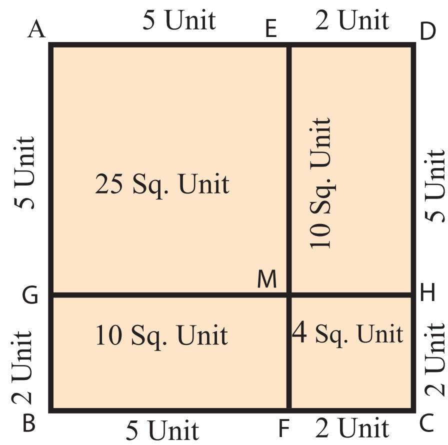


- The number consists of two digits
- The number is larger than half of 100
- It lies between 51 and 75
- Product of the digits of the number lies between 31 and 40.
- Sum of the two digits is 12

- The number consists of two digits
- The number is lesser than half of 100
- Difference of the two digits is 7
- Number in the unitary position is 9
- It is a prime number

Analysis of Mathematical formula or rule

Let us closely observe the following diagram. In the diagram ABCD is a square. The lines EF and GH intersect each other perpendicularly at the point M and divide the square into four parts.



In the Diagram

AB = AG + GB = (5 + 2) units or 7 units,

BC = BF + FC = (5 + 2) units or7 units

CD = CH + HD = (2 + 5) units or 7 units and

AD = AE + ED = (5 + 2) units or 7 units

You already know that area of a square = length of side breadth of side

Area of square ABCD = AB BC = 7 units 7 units or 49 units^2

In the diagram, AGME is a square, where AG = GM = ME = AE = 5 units

Area of square AGME = AG AE = 5 units 5 units or 25 units^2.

In the diagram, CHMF is a square, where CH = HM = MF = FC = 2 units

Area of square CHMF = FC CH = 2 units 2 units = 4 units^2 .

In the diagram, BFMG is a rectangle, whose length BF = 5 units and width BG = 2 units

Area of rectangle BFMG = BF BG = 5 units 2 units or 10 units^2 .

In the diagram, HDEM is a rectangle, whose length HD = 5 units and width DE = 2 units

Area of rectangle HDEM = HD DE = 5 units 2 units or 10 units^2.

Since area of rectangle BFMG = area of rectangle HDEM = 10 units^2 ,

Area of rectangle BFMG + area of rectangle HDEM = 2 area of rectangle BFMG

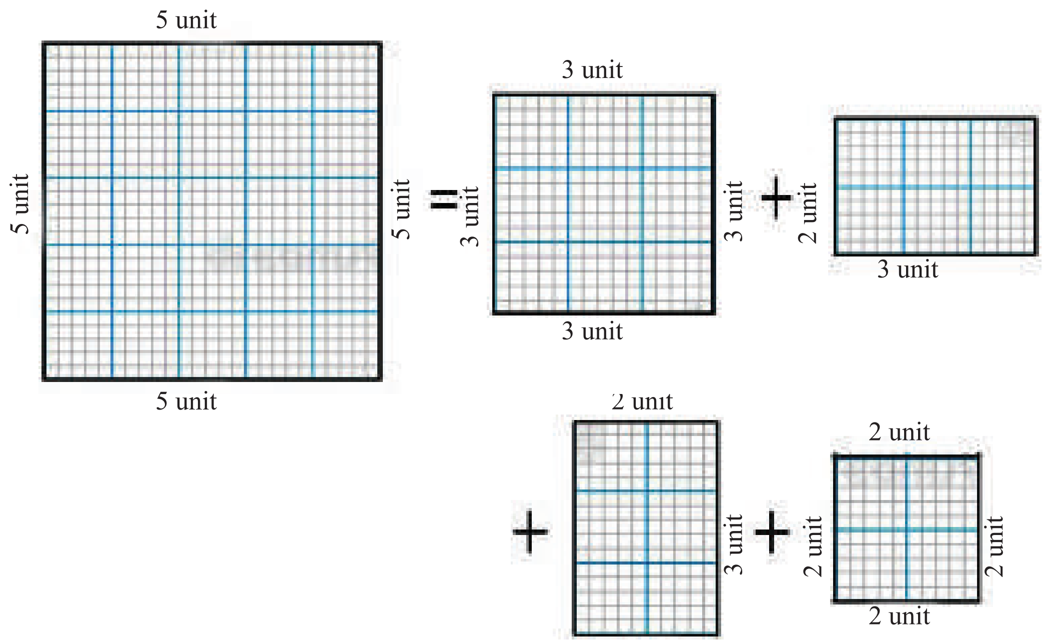
= 2 ×10 units 2 or 20 units^2

Now area of square AGME + area of square CHMF + area of rectangle BFMG + area of rectangle HDEM = (25 + 4 + 10 + 10) = 49 units^2

Hence, we can say that,

Area of square ABCD = area of square AGME + 2 area of rectangle BFMG + area of square CHMF

Verify with paper cutting



Sum of Natural numbers

Now fill up the following table:

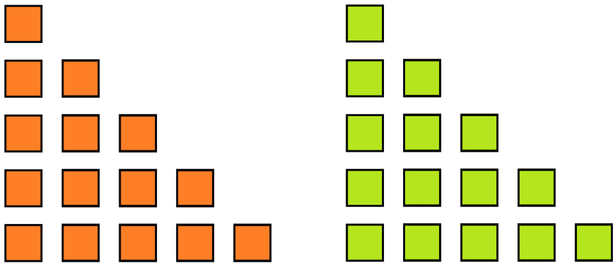
Sum of numbers 1-10	1 + 2 + 3+……………..+10	55
Sum of numbers 1-100	1 + 2 + 3 +……………….+ 100	5050
Sum of numbers 1 – 1000	1 + 2 + 3 +………………+ 1000	500500
Sum of numbers 1 – 10000	1 + 2 + 3+………………..+ 10000	?
Sum of numbers 1 –100000	1 + 2 + 3 +………………..+ 100000	?
Sum of numbers 1 – 1000000	1 + 2 + 3 +.....................+ 1000000	?

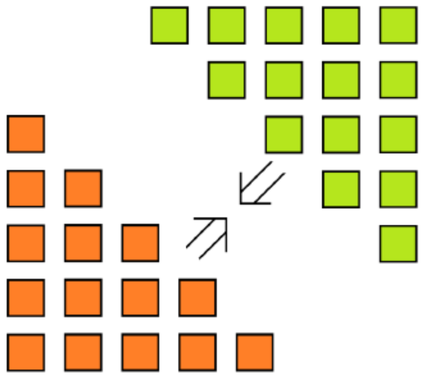
Well, can you find any Mathematical formula or rule in the above table? Now see, if you can find the sum of the numbers from 1 to 50 in using the same rule? Add the numbers consecutively from 1 to 50 and verify the accuracy of the sum obtained according to the rule in the table.

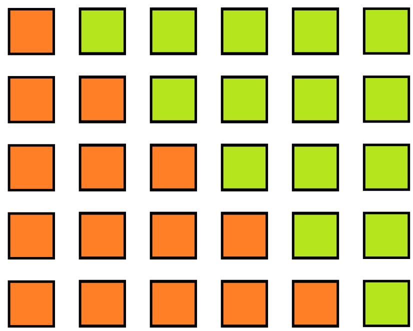
You can understand that the formula or rule of finding the sum of numbers from 1 to 100 is somewhat different from the formula or rule in finding the sum of numbers from 1 to 50.

Then it would have been better if there was a formula or rule with which you could find the sum of natural numbers from 1 to any number.

Alright, let us see if we can find some trick or rule from the following diagrams.







In the last diagram you may find out the number of blocks without counting each one. Can you think of how it can be done? Mind one thing, in the diagram, the number of orange and green blocks are same. Now if you divide or halve the total number of blocks in the last diagram, then you will know the number of orange coloured blocks. Now you have to think, how you can find the sum of the numbers from 1 to 5 in an easier way instead of adding the blocks serially. Can you similarly find the sum of the numbers from 1 to 80? If you want, in a similar way you can find very easily the sum of numbers from 1 to 9000.

Do you know which great Mathematician invented this easy method?

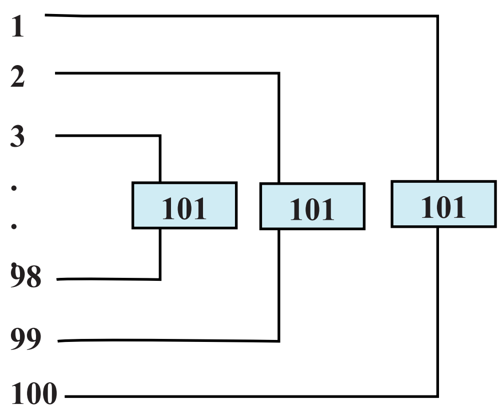
Carl Friedrich Gauss
(1777 – 1855)

He is Carl Friedrich Gauss. Interestingly, he invented this method when he was studying in school like you.

Let us tell that story now.

This is a story of long time ago; Carl Friedrich Gauss was then very young. School teachers used to solve Mathematical problems and puzzles to increase the intelligence of the students and to apply them and justify them. On one such day, Gauss's teacher told (the students) to find the sum of the numbers from 1 – 100. He thought it will take a long time to solve this. Gauss observed everyone in the class was in great distress. Little Gauss thought of a trick. Using a special rule, he found the sum of the numbers 1 – 100 and in a very short time he submitted his work to the teacher. The teacher was surprised that Gauss submitted his work to him before he could even sit down on his chair and relax. The classmates of Gauss were awestruck at this.

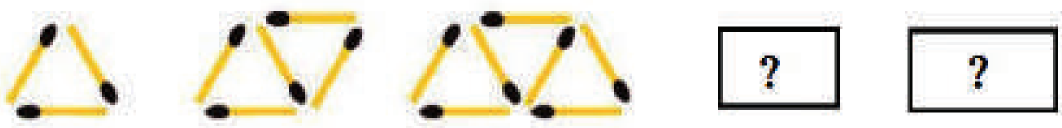
Now you must be wondering how he solved this so easily.
Look at the image below, what was his technique of solution.



Here the first number is 1 and last number is 100, sum of these two is101. Similarly, sum of 2 and 99 is 101, sum of 3 and 98 is 101. Adding in this way, will get 50, 101s. So, you understand easily, the sum of 1 – 100 will be 50 x 101 = 5050. And this is how young Gauss found the sum of numbers from 1 – 100 very easily.

The interesting matter is- from this technique of Gauss, we find an easy Mathematical formula or rule to obtain the sum of natural numbers from 1 to any natural number. Try and see if you find the Mathematical formula or rule?

Individual work: worksheet
Make the designs using matchsticks



1st picture 2nd picture 3rd picture 4th picture 5th picture

a) Make the designs as above using matchsticks.

b) Similarly, using matchsticks of same length make the 4^{th} and 5^{th} designs.

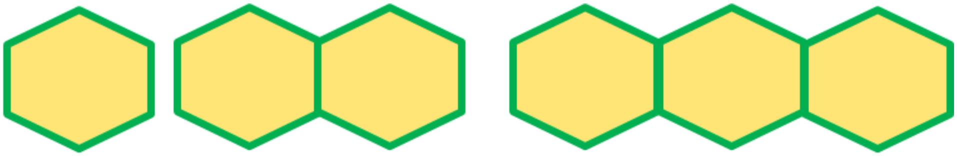
Now fill up the following table:

Picture number	Picture	Number of matchsticks	Mathematical principle
1st			
2nd			
3nd			
4th			
5th			
.			
.			
10th			

c) Express the number of matchsticks required to make the designs in an Algebraic formula.
d) Using the Algebraic formula, find the number of matchsticks required for the 50^{th} design.
e) What will be the total number of matchsticks required to make the 1^{st} 50 designs?

Exercise

1. The geometrical figures below are made up of lines of equal length.



   a) Make the fourth diagram and find the number of lines.

   b) Which mathematical formula or rule is satisfied by the number of lines, explain with logic.

   c) Find out the number of lines required to make the 1^{st} 100 diagrams

2. Anowara Begum saves Tk 500 in the first month from her salary and in the following month she saves Tk 100 more than the previous month.

   a) Express, with explanation, the account of savings with a mathematical formula or rule.

   b) How much does she save on the 30^{th} month?

   c) What are her total savings in the first 3 years?

3. Aurobindu Chakma bought 3 yearly savings certificates with 3 monthly interests, for 5 lac taka from his pension money. The rate of interest is 8% per annum.

   a) Find a Mathematical formula or rule, with explanation, to find the interest.

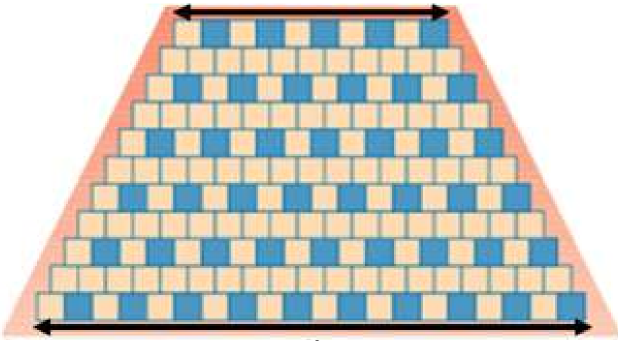
   b) How much interest will he get in the first installment i.e. that is after the first 3 months, use your formula to find that.

   c) How much interest will he get at the end of 3 years?

4. You are told to donate 100 kg rice. But you cannot donate the whole amount at a time. On the 1^{st} day you can donate half of 100 kg, i.e. 50 kg; on 2^{nd} day can donate half of 50 kg, i.e., 25 kg. In this way, every day you must donate half the remaining rice. How many days will you take to donate the entire amount of rice in this way?

   [N.B. you cannot donate less than 1 kg in any way]

5. The following trapezium shaped floor has to be covered by 12 inch square tiles. The number of tiles in each row will be 1 less than the previous row.



a. How many tiles in total will be required to cover the floor?

b. If the cost of tiles is Tk 75 per square feet, how much will be spent for the tiles?

6. A mason/bricklayer got some bricks from a heap of bricks and arranged them in 15 steps. He made two rows in the lowest step and kept 30 bricks on each row.

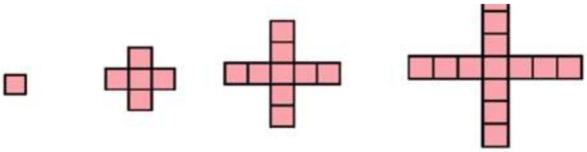
Then for each following step above, he kept 2 less bricks from each row of the step below.

a) How many bricks will be there at the topmost step?

b) Express the process of brick arrangements using a Mathematical formula or rule, with logical explanations.

c) How many bricks has he arranged?

7. Make square tiles of edge 2 cm by cutting paper. Then arrange the tiles as the following diagrams using gum.



a) Make the next diagram

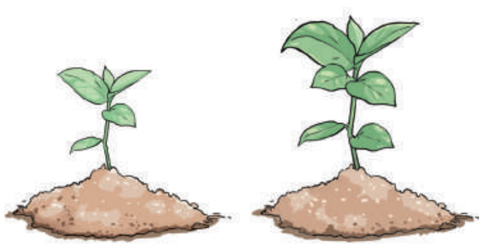
b) Fill up the following table by counting the tiles of each diagram.

Diagram number	1	2	3	4	5	6	.....	10
Number of tiles								

c) Express the number of diagrams and the tiles with a common formula.

d) Draw a line graph using graph paper, taking the diagram number along the x-axis and number of tiles along the y-axis.

8. Mondira sowed 2 sunflower saplings in the courtyard of her home on a Friday. During the sowing, the heights of two plants were 10 cm and 15 cm respectively. She measures the heights of the plants at a fixed time every week. Mondira noticed that the height of the 10 cm high sapling increases 2 cm per week and the 15 cm high sapling increases 1.5 cm every week.



a) Make a list of the increments of heights for two months, of the two saplings from the day they were sown.

b) Express the growth of the two saplings by a Mathematical formula with the definitions of the variables.

c) Draw a line graph taking the weeks along the x-axis and the heights of the two saplings along the y-axis for the data of the first 3 months.

d) Find the point of intersection of the two graphs from the line graph. Explain what is meant by the point of intersection with reference to the two plants.

e) Solving the Mathematical formula obtained in part (b), justify the accuracy of the point of intersection obtained from the graph in part (d).

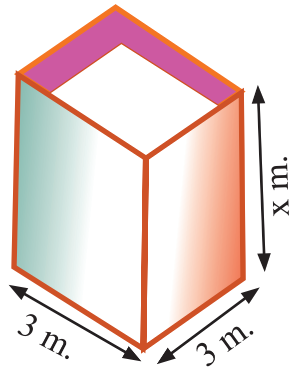
9. The heights of 10 students of class six are as follows (in centimetre) :

Student	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Height (cm)	115	114	122	127	116	x	125	116	117	128

a) If the average height of the students is 120 cm, find the value of x.

b) Find the median and the mode of the heights of the students.

10. The picture is a water tank, whose base is square. Length of the base is 3 metre and the height is x metre.



a) Express the volume V of the tank with a Mathematical formula or rule.

b) Fill up the following table for different values of x:

x	1	2	3	4	5	6	7
V							

c) Draw a line graph using the table obtained in part (b)

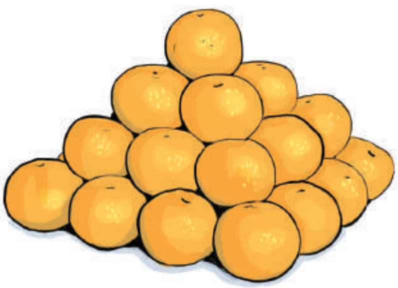
d) What height of the tank will give its volume 15 metre^3?

11. Kamal thought of a three digit number. He gave Shihab few hints to find the number. The hints are:

- The number is less than half of 1212.
- It lies between 502 and 606
- It is not possible to draw a triangle with sides of lengths equal to the three digits of the number.
- The digit in the units place of the number multiplied by the digit in the units place gives a number whose sum of digits will be equal to the digit in the units place.
- The tenth and the units digits are relatively prime.

Like Shihab, you also solve the case of the secret number of Kamal.

12. a) In the picture below, how many oranges are there in the lowest layer?
    b) What is the total number of oranges in the picture?
    c) Have you seen any other fruits or vegetables arranged like this in shops? Find few more examples like this and draw pictures.



উন্নয়নে মৎস্যশিল্প : মাছে-ভাতে বাঙালি

জাতিসংঘের খাদ্য ও কৃষি সংস্থার মতে, সারা বিশ্বে মাছ উৎপাদন বৃদ্ধির হার ৫ শতাংশ হলেও বাংলাদেশে তা ৯ শতাংশ। মাছ উৎপাদন বৃদ্ধির হারে বাংলাদেশ বিশ্বে দ্বিতীয় অবস্থানে রয়েছে। প্রাকৃতিক উৎস থেকে মাছ উৎপাদনে বাংলদেশের অবস্থান বিশ্বে তৃতীয় আর বাংলাদেশের গর্ব ইলিশ উৎপাদনে বাংলাদেশ শীর্ষে। তাই মৎস্য সম্পদ এখন বাংলাদেশের জন্য গর্ব।

Academic Year 2023

Class VI

Mathematics

সমৃদ্ধ বাংলাদেশ গড়ে তোলার জন্য যোগ্যতা অর্জন কর

– মাননীয় প্রধানমন্ত্রী শেখ হাসিনা

জীবে দয়া কর

তথ্য, সেবা ও সামাজিক সমস্যা প্রতিকারের জন্য **‘৩৩৩’** কলসেন্টারে ফোন করুন

নারী ও শিশু নির্যাতনের ঘটনা ঘটলে প্রতিকার ও প্রতিরোধের জন্য ন্যাশনাল হেল্পলাইন সেন্টারে **১০৯** নম্বর-এ (টোল ফ্রি, ২৪ ঘণ্টা সার্ভিস) ফোন করুন

শিক্ষা নিয়ে গড়ব দেশ শেখ হাসিনার বাংলাদেশ

Ministry of Education